MEASURING PRODUCTIVE EFFICIENCY INCORPORATING FIRMS’ HETEROGENEITY: AN EMPIRICAL ANALYSIS

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Heterogeneity among firms is quite prevalent in industries. Using the random coefficients model, this paper aims to measure productive efficiency of firms allowing heterogeneity of firms. Firm level data from the Bangladesh food manufacturing are used for empirical estimation. The results show that there are wide variations in efficiency across firms attributable to firms’ heterogeneity. Further, it shows that there is ample scope for increasing efficiency from the given resources and technology.

Keywords: Productive Efficiency, Frontier Production Function, Heterogeneity of Firms

JEL classification: C22, C32, F14

1. INTRODUCTION

Productive performances of firms in the same industry vary even if they use the identical set of inputs and production technology. Although the neoclassical theory of firm does not attach significance to such differences, empirical studies of production must take into account the differences in evaluating the productive performances of firms; since in the real world, firms are heterogeneous. Firms may differ in the technical and managerial skills available to them. A large number of recent studies provided evidence of firms’ heterogeneity even in narrowly defined industries or markets (Robert and Tybout (1995); Caves (1998); Bartelsman and Doms (2000); Ahn (2001); and Li and Cheng (2005)). Following Knight (1921), economists traditionally view that heterogeneity in association with conduct and performance among firms is quite persistent over time.1 However, some of the factors of firms’ heterogeneity may be

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1 For detail sources of firms’ heterogeneity interested readers are referred to Prahalad and Hamel (1990);
observed but most others are not. So some of these factors can be controlled in the analysis but some cases relevant factors are often complex to be quantified by simple indicators, more so are the factors of unobserved heterogeneity. Failure to recognize the firms’ heterogeneity in the measurement of productive efficiency of firms appears to have lead to misleading conclusions and policy implications.

The seminal work of Farrell (1957) stimulated a large body of both theoretical and empirical research on the measurement of productive efficiency of firms. However, the work of Aigner et al. (1977) and Meeusen and van den Broeck (1977) re-ignited the issue and Stochastic frontier models have become the subject of a voluminous literature. Kumbhakar and Lovell (2000) provide an extensive and excellent survey of this literature. According to them, one of the shortcomings of these conventional frontier models is adjusting the heterogeneity among firms. Although firms’ heterogeneity is a prevalent of industries, the majority of the earlier models either ignore or impose a symmetry restriction for reasons of analytical tractability. A number of recent studies have argued that this simplification may obscure some interesting properties of these models (Kalirajan and Shand (1999), Farsi et al. (2003)). However, some of the earlier studies identified firms’ heterogeneity with random error or inefficiency. If the inefficiency terms are in fact correlated with firm attributes, the estimated parameters and the inefficiency scores from such models will be biased (Tybout (1992)). Therefore, the traditional stochastic frontier techniques may not be suitable for measuring productive efficiency for heterogenous firms. This paper aims to contribute to the economics literature that seeks to measure productive efficiency of firms in the presence of heterogeneity.

The rest of the paper is organized as follows. Section 2 reviewed some earlier literature followed by the analytical framework used in this study. Section 4 presented data and empirical results. Finally, the conclusions are drawn in the last section.

2. CRITICAL REVIEW OF EARLIER LITERATURE

Theoretical models of firm dynamics have formalized the concept of firm heterogeneity and discussed its importance on the measurement of productive performance at firm level. As a result, a good number of studies evolved in order to tackle the heterogeneity while measuring firms’ performance in terms of profitability, returns to scale, efficiency and productivity. For detailed survey, interested readers are referred to Ahn (2001). In his classic study Salter (1969) cites evidence of great heterogeneity in firm performance among plants producing pig iron. Nelson (1991) explained elaborately why firms differ and what matters for that in performance. Fikker and Hasan (1998) estimated production and short-run cost functions in order to estimate

Nelson (1991); Lewin and Phelan (1999); Lall and Rodrigo (2001).
returns to scale in Indian industries. They estimated these functions by transforming the data into first differences on the assumption that unobserved heterogeneity in firms’ productivity levels are no longer part of the error term. However, such estimation exacerbates measurement error problems because such estimators lower the signal to noise ratio (Griliches and Hausman (1986) and Westbrook and Tybout (1993)). Li and Cheng (2005) proposed strategies to model heterogeneity in production units when measuring the structural efficiency of a production group. They classified the production units into different groups and constructed a group of frontier treating these groups separately. Indeed, they introduced extended frontier based on the putty-clay model initiated by Johansen (1970) and Koopmans (1977). However, the authors themselves are not confident on their approach saying that the approach ‘probably not the best solution’ (Li and Cheng (2005), p. 3).

The rapidly growing frontier production function modelling is also concerned with the sources of performance variance of production units. The stochastic frontier model by definition includes a random error term that captures the idiosyncratic heterogeneity among firms. However, as mentioned earlier these studies identified inefficiency with firms’ heterogeneity. Some studies such as Polachek and Yoon (1996) and Greene (2002) extended the traditional stochastic frontier model to panel data by adding a fixed or random effect model. Pitt and Lee (1981) introduced panel data into the stochastic frontier model and interpreted the panel data random effects as inefficiency rather than heterogeneity. Later, Cornwell, et al. (1990) and Battese and Coelli (1992) extended the random effect model to include time-variant inefficiency. Both models have been extensively used in the subsequent studies. However, in both these studies firm-specific effects are treated as inefficiency. Recently, Farsi, et al. (2003) developed a random effect cost frontier model in order to tackle the unobserved heterogeneity among firms. However, their model is very sensitive to empirical data. Therefore, this model cannot be used to developing countries where the availability of reliable firm level cost data are in question.

3. ANALYTICAL FRAMEWORK

Measurement of productive efficiency has received considerable attention from theoretical and applied economists in recent years, particularly after the ‘globalization’ and ‘restructuring’ of many centrally planned and developing economies. However, the available efficiency measures using the conventional stochastic frontier models are not quite sensible, as these models do not take into account the heterogeneity of firms, making the assessment of the success or failure of policy reform difficult. Since the decisions of individual firm are usually independent and management skills differ so the conventional fixed intercept and constant slope frontier models are not appropriate in this case. Therefore, this paper applies the following random coefficient model introduced by Hildreth and Houck (1968) popularised by Swamy (1970), and Kalirajan
and Obwona (1994) in order to develop a framework for measuring productive efficiency allowing heterogeneity among firms. Suppose,

\[
y_i = \alpha \prod_{j=1}^{k} x_{ij}^{\beta_j} \quad i = 1, 2, 3, \ldots, n,
\]

where \( y_i \) and \( x_{ij} \) are \( i \)th firm’s output and \( j \)th input respectively. As it is seen from (1) that the output response coefficients with respect to different inputs vary across firms (implying variation in input application), as do the intercept terms (implying heterogeneity across firms). However, it is important to note that the performance related error is captured by the random coefficients \( \alpha \) and \( \beta \), and that the ‘white noise’ term cannot be distinguished from the random error of the varying intercept term (Hildreth and Houck (1968)). The efficiency indices estimated by using the above model, can be interpreted more consistently with firms’ behaviour and economic theory.

Applying the Cobb-Douglas technology, the random coefficient frontier production function can be written as:

\[
\ln y_i = \beta_j + \sum_{j=1}^{k} \beta_j \ln x_{ij} \quad i = 1, 2, 3, \ldots, n,
\]

where \( y \) refers to output and \( x \)’s are inputs. The above model requires \( nK + n \) coefficients to be estimated with the help of only \( n \) observations. Since intercepts and slope coefficients vary across firms, we can write:

\[
\beta_j = \bar{\beta}_j + u_j,
\]

\[
\beta_j = \bar{\beta}_j + v_j,
\]

where \( \bar{\beta}_j \) is the mean response coefficient of output with respect to \( j \)th input and \( v_j \) is usual random disturbance terms. However, \( u_j \) is a crucial variable in this study, as it reflects the issues relating to heterogeneity of firms that govern the level of output. For example, \( u_j \) captures the institutional and organizational changes due to recent economic reforms. Therefore, if the relation in Equation (2) is obtained by the maximization behaviour of firms, then it is not appropriate to include \( u_j \) additively in Equation (2). Rather, it is appropriate to include \( u_j \) as a determining variable for the parameters of the model (Maddala (1977)) as in Equation (3). This is one of the strong arguments in favour of applying this model in analysing the performance of production units. \( v_j \)’s are random disturbance terms which satisfy all the classical assumptions. In addition to the classical assumptions the following assumptions are also made:
\((\beta_j) = \overline{\beta}_j, \ Var(\beta_j) = \sigma_j^2 > 0, \)

and

\(\text{Cov}(\beta_j, \beta_m) = E(u_j u_m) = 0 \quad j \neq m.\)

These imply that the random coefficients \(\beta_j\)s are independently and identically distributed with fixed mean \(\overline{\beta}_j\) and variance \(\sigma_j^2\).

Combining Equation (2) and (3) one can write

\[
\ln y_i = \overline{\beta}_j + \sum_{j=1}^{\hat{s}} (\overline{\beta}_j + u_j) \ln x_i + v_i. \tag{4}
\]

In matrix format, Equation (4) can be written as

\[
Y = X\beta + D_r u + v, \tag{5}
\]

where \(Y\) is an \((n \times 1)\) vector, \(X\) is a \((n \times K)\) matrix stacked \(x_i'\), the composite disturbance term \(D_r\) is a block-diagonal, with the \(i\)th diagonal block given by \(\Delta = x_i' \Gamma_u x_i\), \(u\) is a \((nK \times 1)\) vector of \(u_i\) s and \(v\) is \((n \times 1)\) vector. It can easily be verified that disturbance terms \(v_i\) and \(u_j\) have zero means and also \(E(v_i u_j) = 0\) for all \(i\) and \(j\), \(E(v_i^2) = \sigma_i^2, E(v_j v_j) = 0\) for \(i \neq j, E(u_i u_i') = \Gamma_u\), a diagonal matrix for \(i = j\) and \(E(u_i u_i') = 0\) for \(i \neq j\).

Given these assumptions, the composite disturbance vector, \(w = D_r u + v\) will have a mean zero and variance \(E(ww') = D_r \Gamma_u D_r + \sigma_i^2 \Gamma_u\).

So, it is apparent that the error structure of the above model violates the basic assumptions of constant variance of the linear regression model, i.e., \(\Delta \neq \sigma^2 I\). The Hildreth-Houck random coefficient model belongs to the class of heteroscedastic error models, where error variances are proportional to the squares of a set of exogenous variables \(x\). So the random coefficient regression model reduces to a model with fixed coefficients, but with heteroscedastic variances. This heteroscedasticity will remain, even if \(\sigma_i^2 = \sigma^2\) values for all \(j\) values so long as the square of the explanatory variables is present. Since the above model is heteroscedastic, the ordinary least squares (OLS) method yields unbiased and consistent but inefficient estimates of mean response coefficients.

The parameters to be estimated are mean response coefficients \(\overline{\beta}\) and variance \(\sigma_i^2\) to obtain the frontier coefficients \(\beta_j\) of the above model. If the elements of \(\Gamma_u\)
are known, the Generalised Least Squares (GLS) technique provides efficient estimators of $\beta_j$ and $\sigma^2_i$. As there is no prior knowledge regarding the elements of $\Gamma_u$, the approach followed in this study to estimate $\Gamma_u$ is one of employed by Swamy (1970) that estimates all the unique elements of $\Gamma_u$ using the OLS. Swamy’s suggested technique is iterative. The initial estimate of $\Gamma_u$ is the identity matrix, used to obtain GLS estimates. Their residuals are also computed and are used to reestimate $\Gamma_u$. This process is repeated using the squared residuals and a new $\Gamma_u$ continues to be estimated until the estimates of $\beta$s are stabilized, i.e., the estimates of $\beta$s and $\Gamma_u$ do not change in repeated iterations.\(^2\) The frontier coefficients $\beta$’s are chosen in such a way to reflect the condition that represent the production responses of following the ‘best practice’ techniques.

Drawing on Kalirajan and Obwona (1994) the assumptions underlying the above model (5) are as follows. The maximum possible output stems from two sources. Firstly, the efficient use of each input contributes individually to the potential output, and can be measured by the magnitude of the varying random slope coefficients ($\beta$ coefficients). Secondly, when all the inputs are used efficiently, then it may produce a combined contribution over and above the individual contributions. This latter ‘lump sum’ contribution, if any, can be measured by the varying random intercept term. The highest magnitude of each response coefficient, and the intercept term from the production coefficients of Equation (5), constitute the production coefficients of the frontier function, showing the maximum possible output.

To elaborate, let $\beta_1^*, \beta_2^*, \beta_3^*, \ldots, \beta_K^*$, be the estimates of the parameters of the frontier production function yielding the potential output. The frontier coefficients $\beta$’s are chosen in such a way to reflect the condition that represent the production responses of following the ‘best practice’ techniques. These are obtained from among the individual response coefficients, which vary across observation as follows:

$$\beta_j^* = \max_{i=1,2,3,\ldots,n} \{\beta_{ji}\} \quad j = 1, 2, 3, \ldots, K.$$  \hspace{1cm} (6)

The key points to note here are first, that these frontier coefficients need not necessarily correspond to the response coefficients for any single individual observation. They may represent the best combination of response coefficients derived from different individual observations. For example, $\beta_1^*$ may come from the 7th observation while $\beta_4^*$ may come from the 16th observation, and so on. This supports the earlier assertion

\(^2\)For further information about this procedure, interested readers are referred to Swamy (1970), Swamy and Mehta (1975) and Griffiths (1972).
that not all farms use each input efficiently. Second, the possibility of obtaining all $\beta_i$'s from a single observation cannot be ruled out. Human capital theory literature argues that a farm which uses some inputs efficiently may also use all inputs efficiently (Kalirajan and Obwona (1994)).

When the response coefficients are estimated then the potential output for the $i$th firm can be worked out as:

$$y_i^* = \sum \beta_j \ln x_{ij},$$

(7)

where $x_{ij}$'s refer to actual levels of inputs used by the $i$th firm. Subsequently, the measure of productive efficiency can be defined as follows:

$$PE_i = \frac{Actual \ output}{potential \ output} = \frac{y_i}{y_i^*}.$$  

(8)

PE varies between 0 to 1. Thus, the random coefficient regression model provides a realistic approach for estimating PE over a large number of firms using cross-section data.  

4. DATA AND THE EMPIRICAL RESULTS

To measure productive efficiency of heterogeneous firms, data came from the Bangladesh food manufacturing industries. Bangladesh Bureau of Statistics (BBS) conducts annual census of industrial production covering firms with more than ten workers. Data on gross output, capital expenditures, energy consumption, and remuneration of workers (all in current prices) are taken for 1999, the last year for which data are released. After dropping observations for which information is either incomplete or where the sum of various components are found to be inconsistent with reported aggregates on the basis of usual accounting principles, 97 firms are selected for this study. Summary statistics of variables are presented in Table 1. At the beginning, the Translog and the Cobb-Douglas specifications for annual data are tested by using the generalized likelihood ratio (LR) test as an important decision-making tool when theoretical considerations do not suggest correct functional specifications. Statistical results support the Cobb-Douglas functional form.  

3 This model can be extended for dealing with panel data. However, it would be most difficult to handle notationally, computationally and analytically.

4 The formal test was conducted to determine the suitable functional form under the null hypothesis is that the coefficients of the cross and squared terms in the Translog function taken together are not significantly
Table 1. Descriptive Statistics of Variables (in Million Taka)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>15.5</td>
<td>55.7</td>
<td>30.4</td>
<td>10.5</td>
</tr>
<tr>
<td>Capital</td>
<td>6.4</td>
<td>35.0</td>
<td>12.5</td>
<td>8.2</td>
</tr>
<tr>
<td>Labour</td>
<td>5.7</td>
<td>18.5</td>
<td>10.4</td>
<td>9.5</td>
</tr>
<tr>
<td>Energy</td>
<td>2.5</td>
<td>12.2</td>
<td>6.5</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Notes: Taka is Bangladesh currency. In 1999 exchange rate was $1US=51.00 Taka.
Source: Calculated using data from the Master Tape of the Census of Manufacturing Industries (CMI) in Bangladesh, Bangladesh Secretariat, Dhaka Bangladesh. Numbers in the parenthesis are standard errors.

Therefore, the following Cobb-Douglas form in the above discussed stochastic coefficients framework is adopted for empirical work:

$$\ln y_i = \ln \alpha_i + \sum_{i=1}^{3} \beta_i \ln x_{i} \quad i = 1, 2, 3, \ldots \ldots \ldots \ldots \ldots \ldots 97,$$

where $y$ refers to gross value of output and $x$’s are capital, labour and energy consumption respectively. The model is estimated using the computer program \textit{TERAN}\textsuperscript{5} and the results are reported in Table 2, which include the mean response coefficients and the frontier coefficients.\textsuperscript{6}

Bruesch-Pagan LM test has been carried out and found to be supported the random coefficient model rejecting the conventional fixed coefficient stochastic frontier models. In addition, actual response coefficients did vary from firm to firm (apparent from Table 2) indicates heterogeneity among firms, which validates the application of random coefficient model in measuring productive efficiency. It also implies that even if firms are using the same inputs but because of the differences in application of inputs contribution of input to output varies from firm to firm. All the mean response coefficients estimated by GLS are statistically significant at the 5 per cent level and signs and magnitudes of these variables are in conformity with theoretical expectations. The estimates of frontier coefficients, presented in the last column of Table 2 indicates different from zero. The calculated $\chi^2_{(n,n)}$ statistics is 1.374 and the log likelihood is 687.542 indicating that the test statistic is not statistically significant.

\textsuperscript{5} \textit{TERAN} was developed in the Division of Economics, Research School of Pacific and Asian Studies, The Australian National University. The program, written in Fortran 77, can be compiled and run on UNIX and VAX based mainframe computers and on IBM PC/AT with 640K memory using Microsoft FORTRAN V.5 and LAHEY FORTRAN V.5.

\textsuperscript{6} The maximum values of the actual response coefficients for each input represent the frontier coefficients.
the maximum possible contribution of inputs to output when firms are operating on their
frontiers and are following ‘best practice techniques’ of given technologies. Moreover,
these estimates are obtained by relaxing the conventional and unrealistic assumption of
symmetry restriction of the coefficients of the frontier production function.

<table>
<thead>
<tr>
<th>Input</th>
<th>Range of Actual Response Coefficients</th>
<th>Mean Response Coefficients</th>
<th>Coefficients of Frontier Production.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.3527-0.4147</td>
<td>0.3547</td>
<td>0.4147</td>
</tr>
<tr>
<td>Capital</td>
<td>0.2574-0.3159</td>
<td>0.2746</td>
<td>0.3159</td>
</tr>
<tr>
<td>Labour</td>
<td>0.2224-0.3614</td>
<td>0.2618</td>
<td>0.3614</td>
</tr>
<tr>
<td>Energy</td>
<td>0.2703-0.3541</td>
<td>0.3904</td>
<td>0.3541</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.0714</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Calculated using data from the Census of Manufacturing Industries (CMI) in Bangladesh, Bangladesh Secretariat, Dhaka Bangladesh. Numbers in the parenthesis are standard errors.

The efficiency estimates are presented in Table 3. There are wide variations in
efficiency rates across firms and sectors of this industry groups. Differences in
production efficiency between firms are generally speaking attributable to firm’s
heterogeneity. Average rates of productive efficiency ranges from 60 to 81 per cent in
different sub-sectors of the Bangladesh food manufacturing industries. The main
implications of this results is that firms in these industries could increase the rate of
productive efficiency by 19 to 39 per cent from the given inputs and technology. In
terms of average rate of productive efficiency, edible oil was the most efficient with 81
per cent mean and a 100 per cent for the most efficient firm in the period studied. This
was followed by rice milling, realizing a mean of about 70 per cent, fruits and vegetables
67 per cent, bakery products 65 per cent and tea and coffee processing 63 per cent
productive efficiency in the same period. However, the average efficiency in the food
manufacturing is more than 68 per cent. So firms of these industries are producing well
below their potentials.

There are not many studies estimated production efficiency using manufacturing
data in Bangladesh. Recently, Samad and Patwary (2002) estimated production
efficiency using the aggregate industry level data. However, they used the traditional
frontier production function ignoring firms’ heterogeneity. Therefore, their estimates fail
to isolate the statistical noise from the measurement of production efficiency. We
suggest caution in accepting their results for policy prescription. Our results are mostly
compatible to some international studies such as Westbrook and Tybout (1993), Roberts and Tybout (1995), and Fikkert nad Hasan (1998) among others in measuring firms’ performance.

Table 3. Productive Efficiency\(^1\) in Bangladesh Food Manufacturing Industries

<table>
<thead>
<tr>
<th>Name of Industries</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bakery Products</td>
<td>56.692</td>
<td>79.835</td>
<td>64.912</td>
</tr>
<tr>
<td>Beverages</td>
<td>48.071</td>
<td>76.223</td>
<td>61.240</td>
</tr>
<tr>
<td>Dairy Products</td>
<td>57.751</td>
<td>74.062</td>
<td>62.406</td>
</tr>
<tr>
<td>Edible Oils</td>
<td>74.069</td>
<td>100.00</td>
<td>81.075</td>
</tr>
<tr>
<td>Fish &amp; Seafood</td>
<td>55.311</td>
<td>69.486</td>
<td>61.527</td>
</tr>
<tr>
<td>Fruits &amp; Vegetables</td>
<td>57.138</td>
<td>78.346</td>
<td>67.145</td>
</tr>
<tr>
<td>Meat &amp; Poultry</td>
<td>52.257</td>
<td>71.969</td>
<td>60.186</td>
</tr>
<tr>
<td>Rice milling</td>
<td>58.049</td>
<td>82.527</td>
<td>70.563</td>
</tr>
<tr>
<td>Tea &amp; Coffee Processing</td>
<td>59.582</td>
<td>77.105</td>
<td>63.347</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>48.311</td>
<td>100.00</td>
<td>68.318</td>
</tr>
</tbody>
</table>

*Source*: Calculated using data from the Census of Industrial Production.

\(^1\) The computer program gives firm-specific productive efficiency (PE). Mean levels of PE in different industries were calculated from firm-specific estimates of all firms in a particular industry and minimum rate is the lowest rate achieved by a firm belong to that industry and like wise the maximum rate.

5. CONCLUSIONS

This paper attempts to estimate productive efficiency applying the principle of random coefficient model in order to allow the heterogeneity of firms. Efficiency estimates derived through this approach are not contaminated with ‘stochastic noise’ unlike the conventional frontier approaches. This method can easily be used with firm-level cross section data, which are often available both in developed and developing countries. In this paper, firm level data from the Bangladesh food manufacturing are used for empirical estimation. Firms in Bangladesh food manufacturing of course are quite heterogeneous as the results show the variability of actual response coefficients (Table 2) and as one considers how firms respond to changes in their economic policies, firm-level heterogeneity may matter. It certainly matters in the Bangladesh case. Thus, firms are producing far away from their potential frontier. The efficiency estimates suggest that there is still scope to increase productive efficiency from the given level of inputs and technology in Bangladesh.
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