DOMESTIC WELFARE EFFECTS OF THE ENTRY OF A FOREIGN FIRM

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The entry of a foreign firm has two counterbalancing effects on domestic social welfare. As the competition level in the domestic market increases by the entry, domestic incumbent firms’ outputs and profits decrease. On the other hand, the price goes down and thus consumers’ surplus increases. Therefore, the effect of the entry of a foreign firm on domestic social welfare is determined by the relative size of these two opposite effects. By investigating this trade-off, we identify domestic market characteristics and types of foreign entrant that are likely to affect domestic social welfare positively. Our main findings can be summarized as follows. First, a foreign firm’s entry is less (more) likely to improve domestic social welfare as the pre-entry overall efficiency level of domestic market is higher (lower). Second, the foreign entrant should be more efficient than domestic firms. Otherwise, domestic social welfare decreases.

Keywords: Domestic Social Welfare, Consumers’ Surplus, Domestic Producers’ Surplus

JEL classification: F12, L10, L13

1. INTRODUCTION

Nowadays our economy becomes more global and there are more tendencies for market opening, import liberalization and entry of foreign firms. For example, in retail and distribution industry, foreign large discount stores have entered the domestic market and we now import foreign cars. Accordingly, we are more interested in the welfare effect of the market opening policy such as entry of foreign firms and the sale of foreign products in domestic market. In general, entry of foreign firms results in the following two counterbalancing welfare effects. First, as the competition level in the market increases by the entry of foreign firms, domestic incumbent firms’ outputs and profits

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decrease, which would be called the ‘crowding-out’ effect. Second, as the market structure approaches more competitive one by the new entry, the equilibrium price decreases and so the consumer surplus increases, which would be called the ‘consumer welfare’ effect. Therefore, the effect of the entry of foreign firms on domestic social welfare is determined by the relative size of the above two opposite forces. Recognizing the fundamental two economic effects of the foreign firm’s entry as above, we investigate the welfare effect of the entry by a foreign firm under various economic circumstances.

Our research is somewhat related with the ‘industrial organization theory’ because we examine the issue of entry. For the case where entrants are domestic firms, there have been numerous researches on the relation between market entry and welfare. At first, the fundamental perceptions of most economists were that the increase in the number of firms raises welfare because of the ‘competition effect’. As time goes on, they found that characteristics of industries and firms could have influence on the welfare implications of entry. For example, von Weizsacker (1980) gets the result that entry of new firms can lower welfare in a market with economies of scale. Perry (1984) and Mankiw and Whinston (1986) show that ‘excessive entry’ relative to social optimum may occur when entry involves fixed costs. Their main logics are as follows. When new entrants enter a market, the total output and consumer welfare increase by the ‘competition effect’. In contrast to this, the output and profit of incumbents go down by the new entry due to ‘business stealing effect’. Also the burden of fixed entry cost is another source of negative welfare effect. In most cases, the negative effects of the latter two are larger than the former positive effect and thus the social optimal number of firms is lower than the equilibrium number of firms. Konishi et al. (1990) get the result that the decrease in the number of firms at free-entry equilibrium can increase welfare in the general equilibrium framework. Klemperer (1988) also shows that new entry can lower welfare when entrants have absolute cost disadvantages compared to incumbents. An obvious observation from these previous researches in ‘industrial organization’ is as follows. Namely, if the entry of a domestic firm reduces domestic welfare for some market, there is no chance for the welfare to increase when a foreign firm with the equivalent technology level enters the same market.

1 Of course, there also is some difference between our research and ‘industrial organization’ because we focus on the entry of foreign firms.

2 Also, Crettez and Fagart (2009) show that increase in the number of firms in some sectors does not necessarily lead to increase in welfare under imperfect competition using general equilibrium model.

3 These researches mentioned here deal with the case of homogenous products. Spence (1976), Dixit and Stiglitz (1977) analyze the case of differentiated products. They also find that there may be too little or too much entry (variety) relative to social optimum.

4 For the case of a foreign firm’s entry, the profit of the entrant is excluded in the domestic welfare, whereas the profit of domestic entrant is included in the domestic welfare.
Our research is also concerned with the ‘international economics’ because we examine the case where an entrant is a foreign firm. In general, the main interest of the traditional international trade theory is to investigate the reason for trade and the trade pattern between two countries. For example, the main result of the traditional theory is that the trade pattern is determined by the factor endowment differences between two countries. This kind of the traditional theory explains mainly the inter-industry trade and it does not analyze the intra-industry trade, which is related with our paper. In order to inquire into the intra-industry trade phenomena, the ‘new trade theory’ has come to the fore. Its main contents are as follows. First, there are more tendencies for intra-industry trade to occur in the monopolistic competition case with product differentiation. And it has been shown that under monopolistic competition, free intra-industry trade yields a higher welfare level than autarky. The gains from trade in this case come from the fact that trade increases the variety of brands facing each consumer in each country. The important researches of this sort are Krugman (1979, 1980) and Helpman and Krugman (1985). These kinds of researches are somewhat different from ours since we look at the homogeneous good cases. Second, when the international market structure is oligopoly rather than perfectly competitive market, intra-industry is likely to occur even if the product is homogeneous. When the market is oligopoly, the pre-trade equilibrium price is above marginal cost, which gives the opportunity for the new foreign product to come into the domestic market and get positive profit. For example, Brander (1981) and Brander and Krugman (1983) explain the intra-industry trade phenomena by applying Cournot oligopoly model. In particular, Brander (1981) is most relevant to our paper, in that both deal with homogeneous product markets within Cournot oligopoly setting and compares the trade case with autarky when the foreign products may come into the domestic market. The main result of his research is that welfare is higher under trade than autarky when the transport costs are sufficiently low. Our paper differs from his in many ways. First, while Brander (1981) looks at the trade (import and export) between countries, our research focuses on the foreign firm’s entry into the domestic market and does not consider the domestic firms’ entry into the foreign market. Second, basically Brander (1981) examines the case where a foreign firm has the same cost as the domestic firm, although the former incurs some transport cost. Our paper, however, investigates also the case where the foreign firm’s marginal cost is lower (or higher) than that of the domestic firm. Third, Brander (1981) does not fully take into account the various pre-trade market structures. To the contrary, our paper gives rich consideration to the characteristics of domestic market such as the number of firms, the distribution pattern of the low and high cost firms, and the efficiency level of domestic firms.

Until now, we have examined previous researches related with entry of new domestic or foreign firms. In our paper, we pursue to derive the welfare effect of a

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5 If the welfare increases under our setting, this positive welfare effect would be enlarged if we include the effect of domestic firm’s entry into the foreign market.
foreign firm’s entry when there are two cost types, those of high marginal cost and low marginal cost, of firms in an oligopolistic domestic market. For this purpose, we adopt a typical Cournot competition model. The main focus of our research is as follows. First, we are interested in the relation between the pre-entry domestic market characteristics (such as the competition level, the number of efficient domestic firms and cost advantage of efficient domestic firms) and the likelihood of domestic welfare increase (or decrease) after the entry of a foreign firm. That is, we would like to determine the domestic market factors that have influences on the direction of domestic welfare change by a foreign firm’s entry. Second, we are concerned with the relation between the efficiency level of a foreign entrant and the domestic welfare change after a foreign firm’s entry. By this analysis, we want to identify which type of a foreign firm is more likely to increase (or decrease) the domestic welfare after the entry. From this kind of research, we hope to derive various domestic market conditions and foreign firm’s type under which it is more likely to increase (or decrease) the domestic welfare after the entry of a foreign firm. The results of this research may be useful as a policy decision criterion whether the deregulation of foreign firm’s entry would be desirable under some economic circumstances. Finally, when the foreign entrant’s marginal cost is much lower than those of domestic incumbents, some of them, in particular inefficient ones, cannot earn positive profits and have to exit from a market. We will also deal with this case and investigate the change of domestic welfare.6

The rest of the paper is organized as follows. In section 2, we introduce a typical Cournot competition model to evaluate the effect of entry of a foreign firm, and derive pre-entry and post-entry equilibrium market configurations. In section 3, we investigate the domestic welfare effect of the entry by comparing them. We also characterize the domestic market conditions and the type of the foreign entrant under which it is more likely to increase (or decrease) the domestic welfare. Section 3 consists of three subsections. As we already mentioned, the domestic market structure changes, depending on the efficiency gap between the foreign firm and domestic firms, i.e., on how much more efficient the foreign firm is than domestic firms. Subsection 1 deals with case when no domestic firms leave the market after the entry. Subsection 2 is the case when inefficient domestic firms exit from the market. Subsection 3 is the case when all domestic firms exit from the market and the foreign firm becomes the monopolist. Section 4 concludes.

6 Since we deal with the case of exit in response to the foreign firm’s entry, this research may be somewhat related with Markusen (2009). But there are big differences in the research purposes: the former is about welfare effect of foreign entrants, the latter is about the decision of foreign entry mode.
2. BASIC MODEL

2.1. Before Entry of a Foreign Firm

Consider a homogeneous product market where \( n \) domestic firms compete in a Cournot fashion. We assume a constant-returns-to-scale technology. We further assume that the marginal cost of each firm is either \( C_L \) or \( C_H \) where \( C_L < C_H \). The number of the firms with the marginal cost of \( C_L \) is \( k \). The marginal cost of the other \( n - k \) firms is \( C_H \). So there are \( k \) efficient firms and \( n - k \) inefficient firms. For simplicity, we assume that there are no fixed costs. We assume that the market demand function is linear and is given by \( p = a - bQ = a - b \sum_{i=1}^{n} q_i \) where \( p \) is the price, \( Q \) is the aggregate output, and \( q_i \) denotes the output of firm \( i \). Without loss of generality, we further assume \( b = 1 \) for simplicity.

Of this typical Cournot model, the equilibrium outcomes are easy to derive. Therefore we will describe them without derivation process. First, let us introduce the notations: Let \( q_i \), \( i = L, H \), denote the equilibrium output of the firm whose marginal cost is \( C_i \), \( \pi_i \) is the equilibrium profit of the firm whose marginal cost is \( C_i \). Let \( Q \), \( CS \) and \( PS \) denote the total output, consumers’ surplus and producers’ surplus (the total sum of profits) at the equilibrium. Social welfare, the sum of \( CS \) and \( PS \), is denoted by \( SW \). We also introduce the notations of \( m = a - C_H \) and \( s = C_H - C_L \) for an expositional purpose. A higher level of \( s \), fixing the level of \( m \), implies a lower level of \( C_L \), or higher efficiency level of the efficient domestic firms. The equilibrium outcomes are as follows.

\[
q_L = \frac{m + (n - k + 1)s}{n + 1},
\]

\[
q_H = \frac{m - ks}{n + 1},
\]

\[
\pi_L = \left[ \frac{m + (n - k + 1)s}{n + 1} \right]^2,
\]

\[
\pi_H = \left[ \frac{m - ks}{n + 1} \right]^2,
\]
\[ Q = kq_L + (n - k)q_K = \frac{nm + ks}{n + 1}, \]

\[ CS = \frac{1}{2} Q^2 = \frac{1}{2} \left( \frac{nm + ks}{n + 1} \right)^2, \]

\[ PS = \frac{nm^2 + 2ksm + [(n + 1)^2 - (n + 2)k]ks^2}{(n + 1)^2}, \]

\[ SW = \frac{n(n + 2)m^2 + 2(n + 2)ksm + [2(n + 1)^2 - (2n + 3)k]ks^2}{2(n + 1)^2}. \]

Notice that we have to assume \( m > ks \) to ensure \( q_{II} = \frac{m - ks}{n + 1} > 0 \). We will keep this assumption of “positive quantity” throughout the paper.

### 2.2. After Entry of a Foreign Firm

Now suppose a foreign firm who produces the homogeneous product enters the domestic market and chooses its quantity simultaneously with \( n \) domestic firms. This is a simple Cournot model with \( n + 1 \) firms, instead of \( n \). Denote the marginal cost of the foreign firm by \( C_F \) and let \( t = C_{II} - C_F \). \( C_F \) may include some transport cost incurred, tariffs, and so on. A higher level of \( t \) denotes higher efficiency level of the foreign firm. If \( t > s \), the foreign firm is more efficient than the efficient domestic firms. If \( t < 0 \), the foreign firm is even less efficient than the inefficient domestic firms. If \( 0 < t < s \), the efficiency level of the foreign firm is between those of the efficient domestic firms and the inefficient domestic firms. The equilibrium outputs of this new Cournot game can easily be obtained as follows:

\[ q_L' = \frac{m + (n - k + 2)s - t}{n + 2}, \]

\[ q_{II}' = \frac{m - ks - t}{n + 2}, \]

\[ q_F = \frac{m - ks + (n + 1)t}{n + 2}. \]
We will assume \(-\frac{m-ks}{n+1} < t < m-ks\) for the present to ensure the “positive quantity” constraints \(q'_{L} = \frac{m-ks-t}{n+2} > 0\) and \(q'_{F} = \frac{m-ks+(n+1)t}{n+2} > 0\), in addition to our previous assumption of \(m > ks\). If \(t < -\frac{m-ks}{n+1}\), the foreign firm will not enter the domestic market since its output and profit after the entry will be 0. Then the equilibrium will remain the same as before. If \(t > m-ks\), it is optimal for the inefficient domestic firms to produce nothing and to leave the market. In this case, there will remain \(k\) efficient domestic firms and the foreign firm in the market. We will return to this case in Section 3.2. and 3.3.

Under the assumptions of \(m > ks\) and \(-\frac{m-ks}{n+1} < t < m-ks\), the corresponding equilibrium outcomes such as each firm’s profit, consumers’ surplus, and domestic producers’ surplus (the total sum of the profits of domestic firms) are also as follows.

\[
\pi'_{L} = \left[\frac{m+(n-k+2)s-t}{n+2}\right]^2,
\]
\[
\pi'_{H} = \left[\frac{m-ks-t}{n+2}\right]^2,
\]
\[
\pi'_{F} = \left[\frac{m-ks+(n+1)t}{n+2}\right]^2,
\]
\[
CS' = \frac{1}{2}(Q')^2 = \frac{1}{2}\left[\frac{(n+1)m+ks+t}{n+2}\right]^2,
\]
\[
PS' = k\pi'_{L} + (n-k)\pi'_{H} = \frac{nt^2 - 2(nm+2ks)t + nm^2 + 4ksm + [(n+2)^2 - (n+4)k]ks^2}{(n+2)^2}.
\]

Then domestic social welfare, the sum of consumers’ surplus and domestic producers’ surplus, is
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3.1. The Case of No Exits of Domestic Firms

Now let us investigate the effects of entry of a foreign firm on consumers’ surplus, domestic producers’ surplus, and domestic social welfare. In this subsection, we will assume the “positive quantity” constraints, 
\[ m > k_s \quad \text{and} \quad \frac{m - k_s}{n + 1} < t < m - k_s, \]
so that no firms leave the market after the entry of a foreign firm.

Then the change of consumers’ surplus due to entry of the foreign firm is

\[
\Delta CS = CS' - CS = \frac{[(2n^2 + 4n + 1)m + (3 + 2n)ks + (n + 1)t][m - ks + (n + 1)t]}{2(n + 1)^2(n + 2)^2} > 0.
\]

Notice that the sign of \( \Delta CS \) is positive since the sign of \( m - ks + (n + 1)t \) is positive by the assumptions of \( m > ks \) and \( \frac{m - ks}{n + 1} < t < m - ks \). This result of enhanced consumers’ surplus is quite obvious because the entry will increase total outputs and will lower the price. For the same reason it is not surprising to see that

\[
\frac{\partial \Delta CS}{\partial t} = \frac{(n + 1)m + ks + t > 0}, \]

which implies that the effect of enhancing consumers’ surplus increases as the foreign firm is more efficient. Let us call this effect of enhancing consumers’ surplus simply the “consumer welfare” effect hereafter.

On the other hand, the sign of the change of domestic producers’ surplus is negative as

\[
\Delta PS = PS' - PS = -\frac{[(2n + 3)nm + (3n + 4)ks - n(n + 1)t][m + (n + 1)t - ks]}{(n + 1)^2(n + 2)^2}.
\]
The first inequality comes from the assumption of \( t < m - ks \). The reason for the decrease in domestic producers’ surplus is because the output for each domestic firm decreases and the price goes down after the entry of the foreign firm. Let us call this effect of decreased domestic producers’ surplus the “crowding-out” effect, in a sense that entry of a foreign firm crows out domestic firms’ outputs. Observe that

\[
\frac{\partial \Delta PS}{\partial t} = -\frac{2(mn + 2ks - nt)}{(n + 2)^2} < 0.
\]

Hence the crowding-out effect increases as the foreign firm gets more efficient. This is because the foreign firm’s output and profit increases, while domestic firms’ outputs and profits decrease as \( t \) grows.

Therefore a foreign firm’s entry has two counterbalancing effects on domestic social welfare: the consumer welfare effect and the crowding-out effect. The net effect on domestic social welfare is determined by the relative magnitudes of these two. To be more concrete, the change of domestic social welfare due to entry of the foreign firm is

\[
\Delta SW = \Delta CS + \Delta PS = \frac{((n + 1)(2n + 1)t - (4n + 5)ks - (2n^2 + 2n - 1)m)[m + (n + 1)t - ks]}{(n + 1)^2(n + 2)^2}.
\]

The sign of \( \Delta SW \) is equal to that of \( (n + 1)(2n + 1)t - (4n + 5)ks - (2n^2 + 2n - 1)m \) as the sign of the term \( m - ks + (n + 1)t \) is positive by the non-negativity constraints. Thus the condition for \( \Delta SW > 0 \), along with the “positive quantity” constraints, is

\[
\frac{(2n^2 + 2n - 1)m + (4n + 5)ks}{(n + 1)(2n + 1)} < t < m - ks.
\]

Notice that this inequality is valid only if \( m > (2n + 3)ks \). If \( m \leq (2n + 3)ks \), there is no range of \( t \) where \( \Delta SW > 0 \) holds. That is, domestic social welfare will decrease as result of the entry of a foreign firm.

Summarizing these results, we have the following proposition.

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7 We can easily verify that \( \Delta q_L = q_L' - q_L < 0 \), \( \Delta q_H = q_H' - q_H < 0 \).

8 Notice that \( m - ks = \frac{(2n^2 + 2n - 1)m + (4n + 5)ks}{(n + 1)(2n + 1)} = \frac{[m - (2n + 3)ks][n + 2]}{(n + 1)(2n + 1)} \).

9 When \( k = 0 \), the inequality \( m > (2n + 3)ks \) is automatically satisfied.
**Proposition 1:** Consider the case where no firms leave the market after the entry of a foreign firm. The entry of a foreign firm will increase consumers’ surplus, but will decrease domestic producers’ surplus. The change of domestic social welfare is as follows.

1. If \( m \leq (2n + 3)ks \), domestic social welfare will decrease after the entry.
2. If \( m > (2n + 3)ks \), there exists \( t' = \frac{(2n^2 + 2n - 1)m + (4n + 5)ks}{(n + 1)(2n + 1)} \) such that domestic social surplus decreases for \( t \in \left( \frac{m - ks}{n + 1}, t' \right) \), but it increases for \( t \in (t', m - ks) \).
3. In particular, we have \( t' > s \). That is, domestic social welfare will decrease if \( t = s \).

**Proof:** The proofs for (1) and (2) are provided in the main text. The proof for (3) is as follows. When \( k = 0 \), (3) is obvious since all the domestic firms are inefficient and thus \( s = c_L - c_L = 0 \). Suppose now that \( k \geq 1 \). Then

\[
t' - s = \frac{(2n^2 + 2n - 1)m + (4n + 5)ks}{(n + 1)(2n + 1)} - s
= \frac{(2n^2 + 2n - 1)m + [(4n + 5)k - (n + 1)(2n + 1)s]}{(n + 1)(2n + 1)}
> \frac{(2n^2 + 2n - 1)ks + [(4n + 5)k - (n + 1)(2n + 1)s]}{(n + 1)(2n + 1)}
= \frac{2kn - 2n + 4k - 1}{2n + 1} > 0.
\]

The first inequality comes from the assumption of \( m > ks \), and the second inequality follows from the supposition that \( k \geq 1 \). Q.E.D.

The condition for domestic social welfare to increase, \( m > (2n + 3)ks \) and \( t > t' \), has two interesting implications. First of all, as the condition of \( m > (2n + 3)ks \) suggests, the number of the efficient domestic firms \( k \), the efficiency level of the efficient domestic firms \( s \), and the competition level of domestic market \( n \) (the number of domestic firms) should be sufficiently low in order for a foreign firm’s entry to improve domestic social welfare. Putting it differently, a foreign firm’s entry is less (more) likely to improve domestic social welfare as the pre-entry overall efficiency level of domestic market is higher (lower). The subsequent Corollary 1, along with the
intuition to be provided, will invigorate this point.

Secondly, the efficiency level of the foreign firm should be sufficiently high, i.e., $t > t'$, to ensure improvement of domestic social welfare. In particular, as Proposition 1-(3) points out, the efficiency level of the foreign firm should be sufficiently higher than that of even the efficient domestic firms, in order for domestic social welfare to improve. That is, domestic social welfare decreases if the efficiency level of the foreign firm is lower than or equal to that of the efficient domestic firms. As we already mentioned, the net effect on domestic social welfare is determined by the relative magnitudes of the two counterbalancing effects: the consumer welfare effect and the crowding-out effect. In this sense, the latter dominates the former when $t$ is small. However, the situation is reversed as $t$ gets larger. This is because the former effect grows more rapidly than the latter as $t$ increases. This can be confirmed from the fact that $\frac{\partial^2 \Delta CS}{\partial t^2} > 0$ and $\frac{\partial^2 \Delta PS}{\partial t^2} < 0$. The reason for the difference in the speed of the two effects can be explained as follows. Consider two price levels, say $p_1$ and $p_2$ with $p_1 > p_2$. Denote by $Q$ total outputs when the price is $p_1$. Suppose now a hypothetical situation where both prices reduce by the same amount, say by $\Delta p$. Then it is easy to see that the increase in consumers’ surplus is larger at $p_2$ than at $p_1$. This is because more consumers will benefit from the price cut when the price is lower and total outputs are larger. Since increase in $t$ reduces the price, the price will be lower when the level of $t$ is higher. Therefore the increase in consumers’ surplus, due to an increase in $t$, is larger when the current level of $t$ is higher and thus the current price is lower. In other words, consumers’ surplus grows most rapidly as $t$ increases.

On the other hand, domestic producers’ surplus decreases less rapidly in $t$. To explain this, let us return to the hypothetical situation where both prices, $p_1$ and $p_2$, reduce by $\Delta p$. In addition, we will assume that domestic firms’ outputs decrease as the price goes down. As a matter of fact, what we have in mind is the situation where domestic firms’ outputs decrease as the price goes down with $t$ increasing. For ease of explanation, assume further that domestic firms are identical, i.e., $s = k = 0$ or $k = n$. Denote by $Q^D$ domestic outputs when the price is $p_1$. Notice that $Q^D_1 > Q^D_2$ by the assumption that domestic outputs are larger at the higher price. Also denote the decrease in domestic outputs, resulted from the price cut of $\Delta p$ at $p_1$, by $Q^D$. Then the reduction in domestic producers’ surplus at $p_1$ is $\Delta p(Q^D_1 - Q^D) + (p_1 - c)\Delta Q^D$, where $c$ is the marginal cost of domestic firms.¹⁰ The first term is the reduction in domestic producers’ surplus due to the reduced price, while the second is the one due to the decreased outputs. It is easy to figure out that the reduction in domestic producers’ surplus is larger at the

¹⁰ Notice that $c = c_H$ when $s = k = 0$, while $c = c_L$ when $k = n$. 
higher price, that is, at $p_1$ than at $p_2$, unless there is too much difference between $\Delta Q_2^D$ and $\Delta Q_1^D$. That is, reductions in domestic producers’ surplus, resulted from a price cut, is larger at higher price, basically because the loss from the price cut is larger when domestic outputs are larger and the current price is higher. Notice now that the situation of the price cut at the higher price, $p_1$, describes the case where $t$ increases when the current level of $t$ is low, while the situation of the price cut at $p_2$ resembles the case where $t$ increases when the current level of $t$ is already high. The above argument shows that the reduction in domestic producers’ surplus, or the size of the crowding-out effect, decreases as $t$ increases.

Corollary 1 provides an interesting comparative statics.

**Corollary 1**: Assume that $m > (2n + 3)ks$.

1. If $k$ or $s$ increases, the range of $t$ where $\Delta SW > 0$ diminishes.
2. If $n$ increases with fixing the ratio of the efficient domestic firms $\alpha = \frac{k}{n}$, the range of $t$ where $\Delta SW > 0$ diminishes.

**Proof**: The proof for (1) is obvious from the fact that $\frac{\partial t'}{\partial k} = \frac{\partial t'}{\partial s} > 0$. To prove (2), denote the ratio of the efficient domestic firms by $\alpha = \frac{k}{n}$. Then $t'$ can be rewritten as

$$
\alpha = \frac{(2n^2 + 2n - 1)m + (4n + 5)ans}{(n + 1)(2n + 1)}
$$

It is easy to verify $\frac{\partial t'(\alpha, n)}{\partial n} > 0$. Q.E.D.

Corollary 1, along with Proposition 1-(1), confirms the implication that a foreign firm’s entry is less (more) likely to improve domestic social welfare as the pre-entry overall efficiency level of domestic market is higher (lower). The intuitions are as

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11 In fact, we have $\Delta Q_1^D = \Delta Q_2^D$ in our model with linear demand function. To see this, observe first that the post-entry equilibrium domestic outputs and price are given by $\frac{nm + 2ks - nt}{n + 2}$ and $\frac{m - ks - t}{n + 2}$, respectively. Now let $t_1$ and $t_2$, with $t_1 < t_2$, denote two distinctive levels of $t$. Also let the corresponding equilibrium prices $p_i = \frac{m - ks - t_i}{n + 2}$, $\alpha = \frac{k}{n}$, denote the two price levels in the main text. Then $\Delta p_i$, the same amount of the price cut at $p_1$ and $p_2$, can be attained by increasing $t_1$ and $t_2$ by the same amount, say $\Delta t$. It is now easy to see from the equilibrium domestic outputs $Q_i^D = \frac{nm + 2ks - nt_i}{n + 2}$ that $\Delta Q_1^D = \Delta Q_2^D = \frac{n\Delta t}{n + 2}$.
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follows. Suppose first that \( n \) increases with fixing the ratio of the efficient domestic firms \( \frac{k}{n} \). An increase in \( n \) means that there are more domestic firms whose outputs and profits are to decrease after the entry of a foreign firm. Thus, the negative impact on domestic firms, or the crowding-out effect, increases as \( n \) increases. On the other hand, the consumer welfare effect gets smaller as \( n \) increases, because the pre-entry level of consumers’ surplus is already high when \( n \) is large. Therefore it becomes more difficult for the entry to increase domestic social welfare when \( n \) is larger. Now let us explain the effect of increasing \( k \). The entry of a foreign firm decreases both the profits of the efficient domestic firms and those of the inefficient firms. However, the loss in profit is larger with the efficient domestic firms than with the inefficient domestic firms.\(^{12}\) Therefore, the total loss in domestic firms’ profits, or the crowding-out effect, will become larger as the number of the efficient domestic firms, \( k \), increases. The consumer welfare effect, on the other hand, turns out to grow less rapidly in \( k \) than the crowding-out effect. This explains why it is more difficult to improve domestic social welfare when \( k \) is larger. Lastly, notice that the effect of increasing \( s \) is exactly opposite to the effect of increasing \( t \). Recall from Proposition 1-(3) and the discussion following Proposition 1 that the efficiency level of a foreign firm(\( t \)) should be sufficiently larger than the efficiency level of the efficient domestic firms(\( s \)), in order for domestic social welfare to improve. This implies that it is more difficult for the entry of a foreign firm to improve domestic social welfare as the efficient domestic firms get more efficient.

3.2. The Case of Exits of the Inefficient Domestic Firms

In this subsection we assume \( t > m - ks(> 0) \), so that \( n - k \) inefficient firms have to leave the market after the entry of a foreign firm. Then there remain \( k \) efficient firms and one foreign firm in the market and the equilibrium outcomes will be as follows:

\[
q_L^* = \frac{m + 2s - t}{k + 2},
\]

\[
q_H^* = 0,
\]

\[
q_F^* = \frac{m - ks + (k + 1)t}{k + 2},
\]

\(^{12}\) Notice that \( \Delta q_i = (p' - c_i)\Delta q_i + \Delta p \cdot q_i \) for \( i = L, H \). Notice also that \( p' - c_L > p' - c_H \), \( q_L > q_H \), \( \Delta p < 0 \), and \( \Delta q_L = \Delta q_H < 0 \). Therefore we have \( |\Delta \pi_L| > |\Delta \pi_H| \).
\[
Q' = kq'' + q' = \frac{(k + 1)m + ks + t}{k + 2},
\]

\[
\pi^*_L = \left[ \frac{m + 2s - t}{k + 2} \right]^2,
\]

\[
\pi^*_F = \left[ \frac{m - ks + (k + 1)t}{k + 2} \right]^2,
\]

\[
CS^* = \frac{1}{2}(Q')^2 = \frac{1}{2} \left[ \frac{(k + 1)m + ks + t}{k + 2} \right]^2,
\]

\[
PS^* = k\pi^*_L = k \left[ \frac{m + 2s - t}{k + 2} \right]^2,
\]

\[
SW^* = CS^* + PS^* = \frac{(2k + 1)t^2 - 2[(k - 1)m + 3ks]t + (k^2 + 4k + 1)m^2 + 2(5 + k)ks + (8 + k)ks^2}{2(k + 2)^2}.
\]

Notice that the equilibrium outcomes in this case have the form of “n” in the expressions of the equilibrium outcomes in the case of no exit being replaced by “k”. In this subsection, we will assume \( t < m + 2s \) to ensure \( q'' = \frac{m + 2s - t}{k + 2} > 0 \), in addition to \( t > m - ks \). If this assumption is violated, only the foreign firm exists in the market and will be the monopolist. We will discuss this case in the next subsection.

The changes of consumers’ surplus and domestic producers’ surplus are the same as in the case of no exit. Consumers’ surplus increases after the entry since

\[
\Delta CS = CS^* - CS
\]

\[
= \frac{(3n + 2kn + k + 1)m + (n + k + 3)ks + (n + 1)t}{2(n + 1)^2(k + 2)^2}[(n + 1)t - (n - k - 1)(m - ks)]
\]

\[
> \frac{(3n + 2kn + k + 1)m + (n + k + 3)ks + (n + 1)t}{2(n + 1)^2(k + 2)^2}[(n + 1)(m - ks) - (n - k - 1)(m - ks)]
\]

\[
= \frac{(3n + 2kn + k + 1)m + (n + k + 3)ks + (n + 1)t}{2(n + 1)^2(k + 2)^2}[(k + 2)(m - ks)] > 0.
\]
The first and the last inequality come from the assumptions of \( t > m - ks \) and \( m - ks > 0 \), respectively. It is interesting to notice that total output increases despite exits of the inefficient domestic firms.

Domestic producers’ surplus, on the other hand, will decrease since

\[
\Delta PS = PS^* - PS
\]

\[
= \frac{(n+1)^2 k t^2 - 2(m + 2s)(n+1)^2 k t + (m - ks)A}{(n+1)^2 (k + 2)^2}
\]

> \[
\frac{(n+1)^2 k(m - ks)^2 - 2(m + 2s)(n+1)^2 k(m - ks) + (m - ks)A}{(n+1)^2 (k + 2)^2}
\]

\[
= \frac{[nm + (n+2)ks][m - ks]}{(n+1)^2} < 0,
\]

where

\[
A = [kn^2 - (k^2 + 2k + 4)n + k]m + [(k + 4)n^2 - (k^2 + 2k - 4)n - 2k^2 - 7k - 4]ks.
\]

The first inequality comes from the fact that \( \Delta PS \) is decreasing in \( t \in (m - ks, m + 2s) \).\(^{13}\)

The change of domestic social welfare is

\[
\Delta SW = SW^* - SW
\]

\[
= \frac{(2k + 1)(n+1)^2 t^2 - 2[(k - 1)m + 3ks](n+1)^2 t + (m - ks)B}{2(n+1)^2 (k + 2)^2},
\]

where

\[
B = -[3n^2 + 6n - k^2 - 4k - 1]m + [(2k + 7)n^2 - 2(k - 1)(k + 3)n - 3k^2 - 10k - 5]ks.
\]

Investigating the sign of \( \Delta SW \) provides us Proposition 2.

**Proposition 2:** Consider the case where the inefficient domestic firms leave the market after the entry of a foreign firm. The entry of a foreign firm will increase consumers’ surplus, but will decrease domestic producers’ surplus. The change of

\(^{13}\) Notice that \( \frac{\partial \Delta PS}{\partial t} = \frac{-2(n+1)^2 k(m + 2s - t)}{(n+1)^2 (k + 2)^2} < 0 \) under the assumption of \( t < m + 2s \).
domestic social surplus is as follows.

1. If \( m > (2n + 3)ks \), domestic social welfare will increase after the entry.
2. If \( m < (2n + 3)ks \), there exists \( t'' \in (m - ks, m + 2s) \) such that domestic social surplus decreases for \( t \in (m - ks, t'') \) and increases for \( t \in (t'', m + 2s) \).

Here \( t'' = \frac{(n + 1)[(k - 1)m + 3ks] + (k + 2)\sqrt{C}}{(n + 1)(2k + 1)} \),

where

\[
C = (n^2 + 2n - 2k)m^2 - 2(2n^2 - 2kn + 3n - 4k)ms + (4n^2 - 4k^2 + 6n - 6k + 1)k^2 s^2.
\]

3. \( t'' \), the lower bound of \( t \) for \( \Delta SW > 0 \), is larger than \( s \).

**Proof**: To prove (1), observe first that

\[
\frac{\partial \Delta SW}{\partial t} = \frac{2(k + 1)t - (k - 1)m - 3ks}{(k + 2)^2} > \frac{2(k + 1)(m - ks) - (k - 1)m - 3ks}{(k + 2)^2} = \frac{m - 2ks}{k + 2} > 0.
\]

The first inequality comes from the assumption of \( t > m - ks \) while the last one from the supposition of \( m > (2n + 3)ks \). Now evaluating \( \Delta SW \) at \( t = m - ks \),

we have \( \Delta SW |_{t=m-ks} = \frac{[m - ks][m - (2n + 3)ks]}{2(n + 1)^2} > 0 \).

This establishes that \( \Delta SW > 0 \) if \( m > (2n + 3)ks \).

Now suppose \( m < (2n + 3)ks \). Since the sign of denominator of \( \Delta SW \), \( 2(n + 1)^2(k + 2)^2 \), is positive, the sign of \( \Delta SW \) is equal to that of the numerator. Denote the numerator by \( N \). Notice \( N > 0 \) is a quadratic inequality with respect to \( t \). Solving this quadratic inequality, we have

\[14\] This inequality is valid only if \( k \geq 1 \).
DOMESTIC WELFARE EFFECTS OF ENTRY OF A FOREIGN FIRM

\[ t < \frac{(n + 1)[(k - 1)m + 3ks] - (k + 2)\sqrt{C}}{(n + 1)(2k + 1)}, \text{ or } t > \frac{(n + 1)[(k - 1)m + 3ks] + (k + 2)\sqrt{C}}{(n + 1)(2k + 1)}, \]

where

\[ C = (n^2 + 2n - 2k)m^2 - 2(2n^2 - 2kn + 3n - 4k)mks + (4n^2 - 4k^2 + 6n - 6k + 1)k^2s^2. \]

However, under the assumption of \( m < (2n + 3)ks \), we can show that

\[ \frac{(n + 1)[(k - 1)m + 3ks] - (k + 2)\sqrt{C}}{(n + 1)(2k + 1)} < m - ks \]
\[ < \frac{(n + 1)[(k - 1)m + 3ks] + (k + 2)\sqrt{C}}{(n + 1)(2k + 1)} < m + 2s. \]

This completes the proof of (2). Q.E.D.

Proposition 2 has implications similar to those of Proposition 1. When the pre-entry overall efficiency level of domestic market is sufficiently low (i.e., when \( m > (2n + 3)ks \)), a foreign firm’s entry improves domestic social welfare, regardless of its efficiency level(\( t \)). However, when the pre-entry overall efficiency level of domestic market is high, the efficiency level of the foreign firm should be sufficiently larger than that of the efficient domestic firms.

Corollary 2 follows from Proposition 2 or the shape of \( t^* \).

**Corollary 2:** Assume \( ks < m < (2n + 3)ks \).

1. Suppose that \( k \) is sufficiently large. Then the range of \( t \) where \( \Delta SW > 0 \) diminishes as \( k \) or \( s \) increases.
2. Suppose that \( k \) is sufficiently large. If \( n \) increases with fixing the ratio of the efficient domestic firms \( \left( \frac{k}{n} \right) \), the range of \( t \) where \( \Delta SW > 0 \) diminishes.

**Proof:** See the appendix.

The implication of Corollary 2 is also similar to that of Corollary 1. That is, a foreign firm’s entry is less(more) likely to improve domestic social welfare as the pre-entry overall efficiency level of domestic market is higher(lower), even in case of exits of inefficient domestic firms. However, there is a slight difference. We require “\( k \)” to be sufficiently large, compared to \( n \), in Corollary 2, while the statement is true for all \( k \) in
Corollary 1. In other words, when \( k \) is small, it is possible that the range of \( t \) where \( \Delta SW > 0 \) increases in \( k \) and \( s \). We believe that this kind of irregularity is due to fact that the entry of a foreign firm causes too drastic changes in market structure. Too many firms, i.e., \( n - k \) inefficient firms, leave the market when \( k \) is small. On the other hand, when \( k \) is sufficiently large, the arguments made for Corollary 1 continue to hold valid as only a small number of firms leave the market.

3.3. The Case of Exits of All the Domestic Firms

Finally, we will assume \( t > m + 2s \) so that the “positive quantity” constraint of the previous subsection is violated as well. Then all the domestic firms leave the market after the entry of the foreign firm, and it will become the monopolist. Its equilibrium output (i.e., the monopolistic output) is \( q^m_F = \frac{A - c_L}{2} = \frac{m + t}{2} \), along with \( q^m_L = q^m_H = 0 \).

Also the other equilibrium outcomes in this case will have the form of “\( k \)” being replaced by “0” in the expressions of the corresponding equilibrium outcomes of the previous subsection. In particular, domestic producers’ surplus is 0 since no domestic firms are left in the market, while domestic social welfare, which is equal to consumers’ surplus in this case, is \( SW^m = CS^m = \frac{(m + t)^2}{8} \).

The sign of the change of domestic social welfare turns out be positive. Therefore we have the following.

**Proposition 3:** In case where all the domestic firms leave the market after the entry of a foreign firm, domestic producers’ surplus decreases. But domestic social welfare as well as consumers’ surplus improves.

**Proof:** Observe that

\[
\Delta SW = SW^m - SW = \frac{(n + 1)^2 (m + t)^2 - 4[n(n + 2)m^2 + 2(n + 2)mn + 2(n + 1)^2 - (n + 3)k]k^2s^2}{8(n + 1)^2}
\]

\[
> \frac{(n + 1)^2 (m + m + 2s)^2 - 4[n(n + 2)m^2 + 2(n + 2)mn + 2(n + 1)^2 - (n + 3)k]k^2s^2}{8(n + 1)^2}
\]

\[
= \frac{m^2 + 2[2n - 2k + n^2 - kn + 1]m + [(2n + 3)k^2 - 2(n + 1)^2 k + (n + 1)^2]s^2}{2(n + 1)^2}
\]

\[
> \frac{[ks]^2 + 2[2n - 2k + n^2 - kn + 1][ks]s + [(2n + 3)k^2 - 2(n + 1)^2 k + (n + 1)^2]s^2}{2(n + 1)^2}
\]
\[ \frac{s^2}{2} > 0. \]

The first inequality is due to the assumption of \( t > m + 2s \) while the second one comes from the assumption of \( m > ks \) along with the fact that \( 2n - 2k + n^2 - kn + 1 \) is positive. Q.E.D.

4. SUMMARY AND CONCLUSION

Recently as degree of market opening in our economy becomes higher, there are more entries into domestic market by foreign firms. Consequently, we are more interested in the welfare effect of market opening policy. Recognizing this kind of change in economic environment, we have examined how a foreign firm’s entry affects domestic welfare under various economic circumstances. In particular, we have focused on the relation between various domestic market characteristics like market competition level, distribution pattern of firms’ cost types and efficiency level of low cost firms and the welfare effect of a foreign firm’s entry. Also, we have investigated how the efficiency level of a foreign entrant affects the domestic welfare.

Our main findings can be summarized as follows. First, it becomes more difficult for a foreign firm’s entry to increase domestic social welfare as the number of efficient domestic firms increases, the marginal costs of domestic firms decrease, and the total number of domestic firms, with fixing the ratio of efficient domestic firms, increases. That is, a foreign firm’s entry is less (more) likely to improve domestic social welfare as the pre-entry overall efficiency level of domestic market is higher (lower). Second, the marginal cost of the foreign firm should be sufficiently lower compared to that of efficient domestic firms. Otherwise, domestic social welfare decreases.

From this study, we can suggest several policy implications as follows. First, a government had better adopt a more strict policy toward market opening as the initial domestic market structure becomes more competitive since it is less likely to increase welfare after a foreign firm’s entry. By the same token, a government needs to consider more generous market opening policy as the domestic market becomes more concentrated. Second, as domestic firms’ overall production technology level rises by increasing R&D investment, the criterion for market openings needs to be higher. Third, as a foreign entrant’s productive capability in terms of efficiency becomes higher, a more generous market opening policy seems to be desirable.

We end the discussion by listing some limitations of the present article that need to be addressed in further researches. First, in this study, we have examined the case of linear demand function and linear cost function. In the future research, it would be desirable if more general demand function and cost function are considered. For example, it would be better if the future study covers the more diverse production
technologies such as IRS (increasing returns to scale) and DRS (decreasing returns to scale) cases. Second, in this paper, we have looked at Cournot competition for homogeneous products. If the future study enlarges the analysis to the Bertrand competition for differentiated products cases, we can figure out how much our results are sensitive to the model change. Third, we have dealt with only the case where a foreign firm enters a domestic market. Later research may need to include also the case where domestic firms enter a foreign market in order to understand the welfare effect of market opening policy more comprehensively and thoroughly. Finally, we have adopted a partial equilibrium model in this study. If a future research introduces a general equilibrium model, then it could examine how the market inter-dependence effects would affect the welfare effect of a foreign firm’s entry.

APPENDIX

The proof of Corollary 2:

To prove (1), observe first that

\[
\frac{\partial t^*}{\partial s} = \frac{3k}{2k + 1} + \frac{k(k + 2)[-(2n^2 - 2kn + 3n - 4k)m + (4n^2 - 4kn + 6n - 6k + 1)ks]}{(2k + 1)(n + 1)\sqrt{C}},
\]

and

\[
\frac{\partial^2 t^*}{\partial m \partial s} = -\frac{2k^2(k + 2)(n - k)(n + 1)ms}{C\sqrt{C}} < 0,
\]

where

\[C = (n^2 + 2n - 2k)m^2 - 2(2n^2 - 2kn + 3n - 4k)ms + (4n^2 - 4k^2 + 6n - 6k + 1)k^2s^2.
\]

Thus \(\frac{\partial t^*}{\partial s}\) is decreasing in \(m \in [ks, (2n + 3)ks]\) and is minimized at \(m = (2n + 3)ks\).

Evaluating \(\frac{\partial t^*}{\partial s}\) at \(m = (2n + 3)ks\) gives us

\[
\left.\frac{\partial t^*}{\partial s}\right|_{m=(2n+3)ks} = \frac{(2n+3)k - (2n^2 - n - 5)k}{2(n+1)(n+1)}.
\]

This proves that \(\frac{\partial t^*}{\partial s} > 0\) for all \(m \in [ks, (2n + 3)ks]\) if \(k > \frac{2n^2 - n - 5}{2n + 3}\). When
k < \frac{2n^2 - n - 5}{2n + 3}, the sign of \frac{\partial t^*}{\partial s} can be negative if m is close to (2n + 3)ks. However, it is easy to verify that \frac{\partial t^*}{\partial s} > 0 if m is sufficiently close to ks, or k is sufficiently close to \frac{m}{s}.

Investigating the sign of \frac{\partial t^*}{\partial s}, however, is more complicated and tedious. Observe that

\frac{\partial t^*}{\partial s} = \frac{3(m + s)(n + 1)\sqrt{C} - [Dm^2 + Em + F]}{(2k + 1)^2(n + 1)\sqrt{C}},

where

D = 3n^2 + 6n + 2k^2 - k + 2,

E = [2(2k^2 - k + 2)n^2 - (8k^3 + 2k^2 + 11k - 6)n - 16k(k^2 + k + 1)]s,

F = -[8(k^2 + k + 1)n^2 - 6(2k^3 + k^2 - 2)n - 18k^3 - 25k^2 - 6k + 2]ks^2.

When n = 2, k = 1 and n = 3, k = 2, it is not difficult to verify that \frac{\partial t^*}{\partial k} > 0 for m \in [ks, (2n + 3)ks]. Now suppose that n \geq 2 and \frac{2n^2 - n - 5}{2n + 3} < k \leq n - 1.

Notice that the sign of \frac{\partial t^*}{\partial k} is identical to that of its numerator, 3(m + s)(n + 1)\sqrt{C} - [Dm^2 + Em + F]. Let g(m) = 3(m + s)(n + 1)\sqrt{C} - [Dm^2 + Em + F].

15 When n = 3, k = 2 is the smallest integer satisfying k > \frac{2n^2 - n - 5}{2n + 3}.

16 Since ks \leq m \leq (2n + 3)ks, we can let m = (1 - \beta)ks + \beta(2n + 3)ks. Then \frac{\partial t^*}{\partial k}, in case when n = 2, k = 1 and n = 3, k = 2, becomes \frac{2s}{3} \left[3\beta + 1 - \frac{18\beta^2 - 1}{\sqrt{24\beta^2 + 1}}\right] and \frac{s}{25} \left[3(16\beta + 3) - \frac{424\beta^2 - 66\beta - 9}{\sqrt{44\beta^2 + 4\beta + 1}}\right], respectively. It is easy to verify that their signs are positive for all \beta \in [0, 1].
It turns out that \( g(m) \) is concave in \( m \in [ks, (2n+3)ks] \).\(^{17}\) Therefore the minimum of \( g(m) \) is obtained either at \( m = ks \) or at \( m = (2n+3)ks \). It is straightforward to see that \( g(m) \) has a positive value at \( m = ks \), while its sign at \( m = (2n+3)ks \) is positive if and only if \( k > \frac{2n^2 - n - 5}{2n + 3} \). This proves that \( \frac{\partial t^*}{\partial k} > 0 \) for all \( m \in [ks, (2n+3)ks] \) if \( k > \frac{2n^2 - n - 5}{2n + 3} \). When \( k < \frac{2n^2 - n - 5}{2n + 3} \), the sign of \( \frac{\partial t^*}{\partial k} \) is negative for \( m \) close to \((2n+3)ks\). However, if \( m \) is sufficiently close to \( ks \), it is easy to verify that \( Dm^2 + Em + F < 0 \), which proves that \( \frac{\partial t^*}{\partial k} > 0 \) for \( k \) sufficiently close to \( \frac{m}{s} \).

To prove (2), denote the ratio of the efficient domestic firms by \( \alpha = \frac{k}{n} \), and let \( t^* = t^*(k, n) = t^*(\alpha n, n) \). Then \( \frac{dt^*}{dn} \), the total derivative of \( t^* \) with respect to \( n \), becomes

\[
\frac{dt^*}{dn} = \frac{k}{n} \frac{\partial t^*}{\partial k} + \frac{\partial t^*}{\partial n},
\]

where the expression for \( \frac{\partial t^*}{\partial k} \) is given in the last paragraph and

\[
\frac{\partial t^*}{\partial n} = \frac{(k+2)(m-ks)(m-(n+2)ks)}{(n+1)^2 \sqrt{C}}.
\]

Rearranging \( \frac{dt^*}{dn} \), we have

\[
\frac{dt^*}{dn} = \frac{1}{(2k+1)^2 (n+1) \sqrt{C}} \left[ \frac{k}{n} g(m) + \frac{(2k+1)^2 (k+2)(m-ks)(m-(n+2)ks)}{n+1} \right].
\]

Using the same arguments given in the last paragraph, we can indeed show that \( \frac{dt^*}{dn} > 0 \) for all \( m \in [ks, (2n+3)ks] \) if \( k > \frac{2n^2 - n - 5}{2n + 3} \). Suppose now that

\( k < \frac{2n^2 - n - 5}{2n + 3} \). It is tedious, but straightforward to verify that the sign of \( \frac{dt^*}{dn} \), evaluated at \( m = ks \), is positive. This proves that \( \frac{dt^*}{dn} \) for \( k \) sufficiently close to \( \frac{m}{s} \). Q.E.D.

\(^{17}\) To show this, verify first that

\[
g^*(m) = \frac{18k^3 k^2 (2k+1)^2 (n+1)^3 (n-k)^2 [(2n+3)ks - (n+2)m]}{C^2 \sqrt{C}}.
\]

Therefore \( g^*(m) \) is maximized at \( m = \frac{(2n+3)ks}{n+2} \). The remaining task is to show that \( g' \left( \frac{(2n+3)ks}{n+2} \right) < 0 \) for all \( m \in [ks, (2n+3)ks] \) if \( n \geq 4 \) and \( k > \frac{2n^2 - n - 5}{2n + 3} \). Since it is very tedious, we omit the rest of the proof.
REFERENCES


