THE VOLATILITY OF THE WON-DOLLAR EXCHANGE RATE DURING THE 2008-9 CRISIS

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This paper estimates the volatility of the won-dollar exchange rate during the 2008-9 crisis. We find that the volatility increased in September 2008 and decreased in May 2009. The volatility rose gradually for one month and subdued in a similar manner, which implies that the volatility was not governed by any specific event or government policy. The overall changes in the volatility are similar to the movements of the CDS premium. We also find that the UK foreign exchange market experienced a similar pattern of volatility shifts and suffered smaller but longer volatility than the Korean one. The volatility shifts are estimated using a Markov switching GARCH model and a Bayesian method is suggested.

\textit{Keywords}: Bayesian Inference, Markov Switching GARCH Models, Exchange Rate Volatility, Credit Crisis

\textit{JEL classification}: C11, C22, F31

1. INTRODUCTION

The Korean foreign exchange market has experienced two crises during the last two decades, as shown in Figure 1. The exchange rate was managed by the government before the Asian financial crisis in 1997-8. The Korean currency, or the won, depreciated sharply during the crisis and showed a trend of appreciating during the next decade. The foreign exchange market suffered from another foreign capital flight during the global credit crisis in 2008-9. Contrary to the case of the 1997-8 crisis, the 2008-9 credit crisis stemmed from the developed economies. However, the credit crisis led

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many developing countries, including South Korea, to an economic or financial crisis. Many policies were implemented to stabilize the financial and foreign exchange markets in Korea. The Bank of Korea has swiftly lowered the policy rate from 5.25% to 2% for 4 months since October 2008. The government announced the Financial Market Stabilization Measures, which included government warrants of the foreign currency debt of the commercial banks and provision of dollar liquidity to them, on October 19, 2008. The Bank of Korea has also made a 30-billion US dollar swap arrangement with the Federal Reserve on October 30, 2008 and other deals with the People’s Bank of China and the Bank of Japan on December 12, 2008. From October 2008 to February 2009, it provided around 30-billion dollar liquidity to financial institutions which had difficulties in overseas fund raising.

This paper estimates the volatility of the won-dollar exchange rate during the 2008-9 crisis and investigates what determined the volatility. We find that the volatility increased in September 2008 and decreased in May 2009. The volatility rose gradually for one month and subdued in a similar manner, which suggests that the volatility was not governed by any specific event or government policy. The overall changes in the volatility are similar to the movements of the CDS premium. We also find that the dollar-pound exchange rate experienced a similar pattern of volatility shifts to that of the won-dollar exchange rate. But, the UK foreign exchange market suffered smaller but
longer volatility than the Korean one.

Since the Autoregressive Conditional Heteroskedastic (ARCH) model was suggested by Engle (1982), the conditional heteroskedastic models have been updated and developed to analyze the volatility of the financial markets. Researchers, including Bollerslev (1986), generalized the ARCH model to GARCH (generalized ARCH) and its variants, such as IGARCH, GARCH-M, and EGARCH. The models are known to describe well the many features of volatility, such as volatility clustering and the leverage effect. However, as Schwert (1990) and Engle and Mustaffa (1992) show, the GARCH models imply too much persistence in the conditional variance. To overcome this shortcoming, Cai (1994) and Hamilton and Susmel (1994) incorporate the Markov switching component into the ARCH model. Gray (1996) and Dueker (1997) generalize their model to Markov switching GARCH (hereafter, MS-GARCH) models. Marucci (2005) shows that MS-GARCH models perform very well in forecasting the financial volatility. Rapach and Strauss (2008) also finds significant evidence of structural breaks in several exchange rate volatilities.

In this paper, we use a MS-GARCH model to estimate the volatility of the exchange rate. However, we confront an implementation problem when we estimate MS-GARCH models by maximum likelihood estimation. Because we have $\sigma_{i-1}^2$ in the structure of GARCH so that the structure becomes recursive, the likelihood function depends on all the past history of the unobservable state variable. This means that if we have K-state and T-sample size, we need to consider $K^T$ cases to get the likelihood function. It is practically impossible to implement. Hamilton and Susmel (1994) and Cai (1994) use Markov switching ARCH models to avoid this problem. Gray (1996) and Dueker (1997) estimate Markov switching GARCH models by approximating the likelihood function.

In this paper, we show that the problem can be easily dealt with in Bayesian context. In Bayesian inference, we estimate parameters and latent state variables simultaneously in a unified way, which means that we can construct the likelihood function assuming we know the states and the parameters. This structure enables us to construct the likelihood function easily. We suggest a Markov Chain Monte Carlo (MCMC) algorithm for estimating MS-GARCH models. Kaufmann and Fruhwirth-Schnatter (2002) use a Bayesian method to estimate Markov switching ARCH models, so they don’t deal with the above problem.

This paper is organized as follows. We review the previous literature in Section 2. Section 3 sets up a MS-GARCH model and suggests a Bayesian method to estimate it. In Section 4, we apply our method to estimate the volatility of the won-dollar and dollar-pound exchange rates. Section 5 concludes this paper.

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1 See Bollerslev, Chou, and Kroner (1992) for details.
2. PREVIOUS LITERATURE

There is a huge amount of research on exchange rate volatility. One group of papers explore the relation between exchange rate volatility and trade or growth. The theoretic ones on the issue are Clark (1973), Obstfeld and Rogoff (1998), Bacchetta and Van Wincoop (2000), Bergin and Tchakarov (2003), Sercu and Uppal (2003), and Koren and Szeidl (2003). The general conclusion is that there is no clear relationship between exchange rate volatility and trade. Empirical studies are following. Dell’Ariccia (1999) and Rose (2000) find a small but significant effect of exchange rate volatility on trade. Clark et al. (2004) argues that there is little empirical evidence that the exchange rate volatility has a negative effect on trade. Rey (2006) finds significant relationship between exchange rate volatility and exports of Middle Eastern and North Africa countries. Tenreyro (2007) shows that the negative effect is not significant once one considers the endogeneity. Aghion et al. (2009) show that exchange rate volatility reduces productivity growth only for countries with low levels of financial development. Bahmani-Oskooee et al. (2012) use industry-level U.S. and Korea data and find that exchange rate volatility has significant effects on mostly small industries.

Another group of research on exchange rate volatility focuses on the determinants of the volatility. Dominguez (1998) and Frenkel, Pierdzoich, and Stadtmann (2005) show that foreign exchange intervention by central banks generally increases exchange rate volatility. Watanabe and Harada (2006) argue that the Bank of Japan’s intervention reduces only the short-term component of exchange rate volatility. Park and Song (2006) show that verbal intervention of the Bank of Japan increased the volatility of the yen-dollar exchange rate. Hau (2002) shows that exchange rate volatility is smaller when the country is more open to foreign trade. Devereux and Engel (2002) argue that high volatility of exchange rate comes from the fact that exchange rates have little effect on economic variables. This “disconnect” may be due to local currency pricing and incomplete international financial markets. Gali and Monacelli (2005) shows that exchange rate volatility depends on monetary policy and policy that generates more volatility achieves more consumer welfare. Markiewicz (2008) argues that the adoption of inflation targeting by the Bank of England decreased exchange rate volatility because it reduced the uncertainty about interest rates. Frommel, Mende, and Menkhoff (2008) show that order flow (or private information) and news (or public information) account for exchange rate volatility. Chung and Jorda (2009) examine the relationship between the carry trade and exchange rate volatility.

3. MODEL AND ESTIMATION METHOD

A MS-GARCH \((r, s)\) model is

\[
y_t = x_t'y_S + e_t, \quad e_t = \sigma_t\omega_t, \quad \omega_t \sim N(0,1),
\]

\[
\sigma_t^2 = \mu_{S_t} \sum_{j=1}^r \alpha_j e_{t-j}^2 + \sum_{j=1}^s \beta_j \sigma_{t-j}^2,
\]

where \(y_t\) is the dependent variable, \(x_t\) is the vector of the independent variables, \(y = [y_1, y_2]'\) is the regression coefficient vector, \(\gamma_{S_t} = y_1(1-S_t) + y_2 S_t\), and \(\mu_{S_t} = \mu_1(1-S_t) + \mu_2 S_t\). \(\alpha = [\alpha_1, \cdots, \alpha_r]'\) and \(\beta = [\beta_1, \cdots, \beta_s]'\) are the coefficients of the GARCH process; and \(S_t\) is the state variable taking 0 or 1. For model identification, we impose a condition of \(\mu_1 < \mu_2\). The state variable, \(S_t\), evolves according to a two state, first order Markov Switching process with the following transition probabilities:

\[
Pr[S_t = 0| S_{t-1} = 0] = p_{11}, \quad Pr[S_t = 1| S_{t-1} = 1] = p_{22}.
\]

A goal of Bayesian inference is to derive the posterior distributions of the parameters and the state variables conditional on the data. First, we construct the posterior distribution via the Bayes’ rule. The posterior density of the model is

\[
p(\Theta, S | Y) \propto p(\Theta, S)p(Y | \Theta, S)
\]

\[
\propto p(\Theta)p(S | \Theta)p(Y | \Theta, S),
\]

where \(\Theta = (y, \alpha, \beta, \mu_1, \mu_2, p_{11}, p_{22})\), \(S = (S_1, \cdots, S_T)\), and \(Y = (y_1, \cdots, y_T)\). The Bayes’ rule is applied in the first line. The second line is due to the definition of conditional probability.

\(p(\Theta)\) is the prior for the parameters. Under the assumption of independence, the prior density is chosen as

\[\text{To guarantee the positivity of the conditional variance, } \sigma_t^2, \text{ we impose constraints on the GARCH coefficients as follows: } \mu_i > 0, \quad \alpha_i > 0 \quad \text{and} \quad \beta_i > 0 \quad \text{for all } i.\]
\[
p(\Theta) = p(\gamma)p(\mu)p(\alpha)p(\beta)p(p_{11})p(p_{22}) = N(\mu_\gamma, \Sigma_\gamma) \times N(\mu_\mu, \Sigma_\mu) \cdot I_{(\gamma_i > 0, \gamma_i)}
\times N(\mu_\beta, \Sigma_\beta) \cdot I_{(\beta_i > 0, \gamma_i)}
\times \text{Beta}(u_{11}, u_{12}) \times \text{Beta}(u_{22}, u_{23}),
\]

(4)

where \( N(\cdot) \) is the normal density function, \( I_{(\cdot)} \) an indicator function, and \( \text{Beta}(\cdot) \) the beta density function. We obtain \( p(S | \Theta) \) based on the fact that \( p(S | \Theta) = p(S | p_{11}, p_{22}). \)

\[
p(S | \Theta) = p(S | p_{11}, p_{22})
= \prod_{t=1}^{T-1} p(S_{t+1} | S_t, p_{11}, p_{22})
= p_{11}^{\eta_{11}} (1 - p_{11})^{\eta_{12}} p_{22}^{\eta_{22}} (1 - p_{22})^{\eta_{21}},
\]

(5)

where \( \eta_{ij} \) refers to the number of the transitions from state \( i \) to \( j \). The second line is due to the Markov property of \( S \). The likelihood function, \( p(Y | \Theta, S) \), is derived as

\[
p(S | \Theta, S) = \prod_{t=1}^{T} p(y_t | Y_{t-1}, S_{t-1}, S_t, \Theta)
= \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left[ -\frac{y_t^2}{2\sigma_i^2} \right].
\]

(6)

Note that the density of \( y_t \) depends on not only \( S_t \) but also \( (S_{t-1}, \ldots, S_1) \), which are all the past history of the state variable. The reason is that, when \( r = s = 1 \) for example, \( \sigma_i^2 \) in Equation (2) depends on \( S_t \) and \( \sigma_{t-1}^2 \), and \( \sigma_{t-2}^2 \), and so on. In classical inference, it is not easy to handle such cases because the standard Hamilton filter does not work. One way to avoid this problem is to approximate the density of \( y_t \) so that it depends on finite lags of \( S_t \). However, in Bayesian inference, we estimate the parameters and the state variables simultaneously. As a result, we treat \( S \) and \( \Theta \) as known when we construct the likelihood function.

In order to generate \( S_t \), we use the single move procedure suggested by Yoo (2010), who derives the conditional distribution of \( S_t \) when the likelihood function depends on all the past history of \( S_t \). The conditional distribution is obtained as follows:
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\[ p(S_i \mid Y, S_{t-1}, \Theta) \propto p(S_i \mid \Theta) p(Y \mid S_i, \Theta) \]
\[ \propto p(S_{t+1} \mid S_t, \Theta_t) p(S_t \mid S_{t-1}, \Theta_t) \]
\[ p(y_t \mid Y_{t-1}, S_{t-1}, \ldots, S_t, \Theta) \ldots p(y_T \mid Y_{T-1}, S_T, \ldots, S_t, \Theta) \]
\[ \prod_{s=t}^{T} (\sigma_s^2)^{-1/2} \exp \left[ -\frac{1}{2} \sum_{s=t}^{T} \frac{e_s^2}{\sigma_s^2} \right], \quad (t = 1, \ldots, T) \quad (7) \]

We calculate \( \Pr(S_i = 0 \mid Y, S_{t-1}, \Theta) \) and generate a random number from uniform distribution between 0 and 1. If the random number is less than the probability we set \( S_i = 0 \), otherwise \( 1 \). In order to generate the parameters, we use Nakatsuma’s (2000) algorithm.\(^3\)

4. ESTIMATION RESULTS

We estimate the following MS-GARCH (1,1) model using the daily won-dollar exchange rate from January 2008 to November 2009:

\[ y_t = \gamma_0 + \gamma_s y_{t-1} + \varepsilon_t, \quad \varepsilon_t = \sigma_t \omega_t, \quad \omega_t \sim N(0,1), \]
\[ \sigma_t^2 = \mu_{\sigma} + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \]

where \( y_t \) is the log-difference of the exchange rate, \( \gamma_s = \gamma_1 (1 - S_t) + \gamma_2 S_t \), and \( \mu_{\sigma} = \mu_1 (1 - S_t) + \mu_2 S_t \). The first four columns of Table 1 show the posterior means and 95% highest posterior density intervals (HPDIs)\(^4\) of each parameter. The estimates of \( \gamma_s \) are not statistically significant, which means that there is no autocorrelation in the log-differenced exchange rate regardless of the states. Significant regime shifts are detected since the HPDIs of \( \mu_1 \) and \( \mu_2 \) do not overlap. The unconditional variance of the high volatility regime (\( S_t = 1 \)) is about 10 times of that of the low volatility regime (\( S_t = 0 \)).\(^5\)

\(^3\) See Appendix for the details of the MCMC algorithm.
\(^4\) They are also called the highest posterior density credible set in Bauwens, Lubrano and Richard (1999).
\(^5\) The unconditional variance of each regime is \( \frac{\mu_1}{1 - \alpha_1 - \beta_1} = 0.63 \) and \( \frac{\mu_2}{1 - \alpha_1 - \beta_1} = 5.79 \), respectively.
Table 1. Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>Korea mean</th>
<th>95% HPDI</th>
<th>U.K. mean</th>
<th>95% HPDI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_0$</td>
<td>0.0083</td>
<td>[-0.0551, 0.0743]</td>
<td>-0.0246</td>
<td>[-0.095, 0.0416]</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-0.0143</td>
<td>[-0.1497, 0.1103]</td>
<td>-0.1038</td>
<td>[-0.2578, 0.0513]</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>-0.0136</td>
<td>[-0.2103, 0.1487]</td>
<td>0.1727</td>
<td>[0.0373, 0.3065]</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>0.0593</td>
<td>[0.0291, 0.1008]</td>
<td>0.1364</td>
<td>[0.0771, 0.1927]</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>0.5588</td>
<td>[0.2196, 0.8465]</td>
<td>0.5527</td>
<td>[0.303, 0.802]</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.2155</td>
<td>[0.1130, 0.3219]</td>
<td>0.1141</td>
<td>[0.0023, 0.2077]</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.6908</td>
<td>[0.5748, 0.8070]</td>
<td>0.5041</td>
<td>[0.3578, 0.6519]</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>0.9923</td>
<td>[0.9814, 0.9999]</td>
<td>0.9854</td>
<td>[0.9678, 0.9995]</td>
</tr>
<tr>
<td>$p_{22}$</td>
<td>0.9859</td>
<td>[0.9663, 0.9999]</td>
<td>0.984</td>
<td>[0.9656, 0.9998]</td>
</tr>
</tbody>
</table>

Figure 2 shows the estimates of $\hat{S}_t$ or $\tilde{S}_t$, against the exchange rate. The exchange rate volatility increased in September 2008 and decreased in May 2009. One interesting finding here is that the regime shifts didn’t happen abruptly but gradually. For example, $\hat{S}_t$ started to increase in the late August 2008 and reached to 1 in the late September 2008. It took almost one month for the volatility to reach a peak. Notably, the foreign exchange market started to suffer from the volatility even before the Lehman Brothers bankruptcy on September 4th. The same applies to the case of the decrease in the volatility in May 2009. $\tilde{S}_t$ started to fall in the late April 2009 and reached to 0 in the late May 2009. All this implies that the volatility in the exchange rate was not governed by any specific event or policy.

To cope with the foreign exchange market crisis, the Korean government implemented many policies, including Financial Market Stabilization Measures and US Currency Swap Agreement. However, it turns out that these policies could not decrease the volatility substantially. Of course, the currency swap with US was especially effective to pull down the level of the exchange rate at least temporarily as shown in Figure 2. But, it failed to subdue the volatility of the exchange rate. Again, all this confirms our conclusion that the volatility in the exchange rate was not determined by any specific event or policy. This finding is comparable to that of Frank and Hesse (2009), who show that central bank policies in US and Europe had a little impact on the stressed interbank markets and failed to contain the crisis.
During the 2008-9 crisis, Korea also suffered from abrupt foreign capital flight and currency depreciation, as many other emerging economies did. Especially, the foreign exchange market is very sensitive to foreign investors’ behavior on the Korean stock markets. Figure 3 confirms this observation. Before the surge of the exchange rate volatility, there were big sell offs by foreign investors. But, foreign purchases of the Korean stocks couldn’t alleviate the volatility right away. Foreign investors started to buy the Korean stocks in March, 2009 and the volatility didn’t change much until May, 2009.
Similar phenomenon can be found in Figure 4. There was a big deficit in the current account before the volatility rose. After some up and downs, the current account has remained positive since February, 2009. Here again, however, the volatility continued to be high for a few months later.

One variable that might explain the falling of the exchange rate volatility is the credit-default-swap (CDS) premium, which is the cost of insuring against default risk. Figure 5 shows that the timing of the fall of the CDS premium corresponds to that of the volatility. However, we find another puzzle here. When the volatility was raised in September, 2008, the CDS premium didn’t increase much for the time being. It seems that there was a month lag for the premium to spike.

All this implies that psychological factors might play an important role in the foreign exchange market during the crisis. As Eichengreen and Mody (1998) noted, emerging economies tend to be affected by market sentiment, rather than fundamentals, in a depressing period.

When the Korea was hit by capital flight in September, 2008, its fundamentals had been considered sound and the foreign reserves were bigger than all but five other countries’. However, the foreign investors worried about the short-term debt of the banking sector, which led to the sharp depreciation of the won. In fact, the won was more vulnerable than any other Asian currencies at that time.6

The counterpart was the UK among the advanced economies. Even though Britain’s public finances and economic performance were not worse than any other large economies, the pound turned out to be especially vulnerable after the credit crisis as shown in Figure 6. Investors were thought to be too concerned about the size of Britain’s banking liabilities.\(^7\) That is, South Korea and Britain have had similar experience in the sense that the banking sectors of both countries, not the fundamentals, were blamed for the sharp depreciation during the credit crisis.

\(^7\) Refer to “Sinking sterling” in The Economist, Dec 18th 2008.
To compare the exchange market volatilities between South Korea and the UK, we estimate the same model using the dollar-pound exchange rate. Table 1 shows the estimation results. Similar to the case of Korea, there were significant regime shifts in the volatility since the HPDs of $\mu_1$ and $\mu_2$ do not overlap. However, the magnitude is not as big as that of Korea. The unconditional variances of each state in the UK are 0.36 and 1.45 while those of Korea are 0.63 and 5.96. That is, the variance of the dollar-pound exchange rate increased around 3 times during the 2008-9 crisis, which is much more moderate than that of the won-dollar rate.

As Figure 7 shows, the volatility of the dollar-pound exchange rate has shifted up in September 2008 and the timing is similar to that of Korea. However, the high volatility of the dollar-pound exchange rate lasted longer than the won-dollar rate almost by one month. In addition, the UK exchange market suffered another increase in the volatility in September, 2009. All this suggests that the global credit crisis hit the two countries at the same time but the depth or length of the effects were different depending on the states of each economy.

5. CONCLUSIONS

The global credit crisis of 2008-09 stemmed from the advanced economies but led many developing countries, including South Korea, to a crisis in foreign exchange markets. In this paper, we estimate the exchange rate volatility of South Korea and the
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U.K. in order to examine the timing of the foreign exchange market turmoil, any determinant of the volatility shifts, and the difference between the two countries.

The main results are as follows. We find that the volatility increased in September 2008 and decreased in May 2009. The volatility was raised gradually for one month and subdued in a similar manner, which suggests that the volatility was not governed by any specific event or government policy. The overall changes in the volatility are similar to the movements of the CDS premium. We also find that the dollar-pound exchange rate experienced a similar pattern of volatility shifts to that of the won-dollar exchange rate. But, the UK foreign exchange market suffered smaller but longer volatility than the Korean one.

This paper, however, didn’t find any determinant of the recent changes in exchange rate volatility. We just find that a specific event or government policy was not the factor. Based on the fact that the movement of the CDS premium is closely related to exchange rate volatility, we might need to pay attention to the determinants of the CDS premium. In addition, psychological factors were likely to work in the foreign exchange market during the crisis. Those things need to be considered in future research.

APPENDIX

Once we have posterior density function, we get marginal posterior density functions of parameters and state variables by integrating the posterior density function. Markov Chain Monte Carlo (MCMC) is one way of numerical integration. MCMC algorithms are based on the Clifford-Hammersley theorem. The theorem says that a joint distribution can be characterized by its complete conditional distributions. In our context, the posterior distribution, \( p(\Theta, S | Y) \), is characterized by the complete conditional distributions, \( p(\Theta | S, Y) \) and \( p(S | \Theta, Y) \). Given the initial values, \( \Theta^{(0)} \) and \( S^{(0)} \), we draw \( \Theta^{(1)} \) from \( p(\Theta | S^{(0)}, Y) \) and then \( S^{(1)} \) from \( p(S | \Theta^{(1)}, Y) \). Iterating these steps, we finally get \( \{ S^{(g)}, \Theta^{(g)} \}_{g=1}^G \). Under some mild conditions, it is shown that the distribution of the sequence converges to the joint posterior distribution, \( p(\Theta, S | Y) \).

Gibbs samplers and Metropolis-Hastings (MH) algorithms are used for drawing the parameters and the state variables.\(^8\)

For MCMC implementation, we divide the parameters, \( \Theta \), into three categories:

\(^8\) The details of MCMC methods can be found in Robert and Cassela (1999) and Johannes and Polson (2003).
\[ \Theta_1 = (p_{11}, p_{22}), \]
\[ \Theta_2 = (\gamma), \]
\[ \Theta_3 = (\mu, \alpha, \beta). \]

Let \( S_{st} = (S_{1t}, \ldots, S_{t-1}, S_{t+1}, \ldots, S_T) \). Then the MCMC algorithm is summarized as followed:

- Draw \( S_{st}, (t=1, \ldots, T) \) from \( p(S_t | S_{st}, Y, \Theta) \) by the single move procedure,
- Draw \( \Theta_1 \) from \( p(\Theta_1 | S) : Beta \),
- Draw \( \Theta_2 \) from \( p(\Theta_2 | S, Y, \Theta_3) \) by MH,
- Draw \( \Theta_3 \) from \( p(\Theta_3 | S, Y, \Theta_2) \) by MH.

For the MH algorithm for \( \Theta_3 \), we use the following model, as in Nakatsuma (2000),

\[ \epsilon_t^2 = \mu_t + \sum_{j=1}^{l} (\alpha_j + \beta_j) \epsilon_{t-j}^2 + w_t - \sum_{j=1}^{s} \beta_{t-j}, \quad w_t \sim N(0, 2\sigma_t^4), \]

where \( w_t = \epsilon_t^2 - \sigma_t^2 \), \( l = \max\{r, s\} \), \( \alpha_j = 0 \) for \( j > r \), \( \beta_j = 0 \) for \( j > s \) and \( \epsilon_t^2 = 0 \) and \( w_t = 0 \) for \( t \leq 0 \). Then, the corresponding likelihood function is

\[ p(\epsilon^2 | Y, S, \Theta_2, \Theta_3) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi(2\sigma_t^4)}} \exp \left[ - \frac{w_t^2}{2(2\sigma_t^4)} \right], \]

where \( \epsilon^2 = [\epsilon_1^2, \ldots, \epsilon_T^2]^\top. \)

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