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# ENVIRONMENTAL DYNAMICS IN AN INTEGRATED WALRASIAN-GENERAL EQUILIBRIUM AND NEOCLASSICAL-GROWTH THEORY

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This study deals with interactions of economic growth and environmental change with heterogeneous households. The analytical framework is built by integrating the three important theories in economics - the Walrasian general equilibrium theory, the neoclassical growth theory, and the neoclassical growth model with endogenous environment. The three theories are integrated by applying Zhang's approach to household behavior. The economic system consists of one capital goods sector, one consumer goods sector, one environmental sector, and any number (of types) of households. The motion is described by a set of differential equations. For illustration, we simulated the motion of the economic system with three groups. The comparative analyses provide some insights into the complexity of economic growth with environment. For instance, the study by Grossman and Krueger (1995, p. 353) identifies no evidence that "environmental quality deteriorates steadily with economic growth." Our simulation indicates that although the conclusion made by Crossman and Krueger holds for the national economy, but is invalid for a certain group.

*Keywords*: Environment, Taxes on Production and Consumption, An Integrated Walrasian-General-Equilibrium Theory and Neoclassical-Growth Theory, Inequality
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## 1. INTRODUCTION

There is an increasingly complicated interdependence among economic growth, economic structural changes, inequalities in income and wealth, and environmental change over time and space. Moreover, roles of the governments on the complexity of these dynamic interactions are changing rapidly in different parts of the world. To

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properly deal with these complicated interactions, we need to deal with the economic system as an integrated whole rather than separated subsystems. This is particularly true when we explain environmental changes. It is obvious that production and consumption may lead to deterioration environmental quality and bad environment also lower productivity and utilities. Without an integrated framework of economic and environmental systems, we can hardly explain how environment changes over time. The purpose of this study is to provide some insights into a challenging issue pointed out by Lin and Liscow (2012, p. 268): “The effects of increasing income on environmental quality is an issue that has long puzzled economists. For over decade, economists have theorized that a graph of environmental degradation versus income often looks something approximating an inverted-U shape, dubbed the environmental Kuznets curve (EKC) after Simon Kuznets’ work in the 1950s and 1960s on income equality (Kuznets, 1955, 1965).” It is also argued that the significance of this question is that a fundamental understanding of this issue would help societies to have a prosperous economy while preserving the environment.

In recent years environmental issues have received more attention than ever. Tsurumi and Managi (2010) point out three effects that are important in explaining the level of environmental pollution and resource use. The three effects are: (i) increases in output tends to require more inputs and produce more emissions; (ii) changes in income or preferences may lead to policy changes which will affect production and thus emission; and (iii) as income increases, the economic structure may be changed which will causes changes in the environment (see also, Lin and Liscow, 2012). It is argued that the net effect of these effects tends to result in the environmental Kuznets curve, even though a large number of empirical studies find different relations - for instance, inverted U-shaped relationship, a U-shaped relationship, a monotonically increasing or monotonically decreasing relationship - between pollution and rising per capita income levels (e.g., Bravo and Marelli, 2007; Tsurumi and Managi, 2010). Another important aspect that has been often neglected in the literature of formal growth models with environment is related to heterogeneous households. Homogenous population is often assumed. This assumption should be relaxed. For instance, it has been empirically confirmed that poor and rich people are different in their concerns about environmental degradation and willing to pay for its prevention. As observed by Fairbrother (2013, p. 910), “Recent survey research argues that richer people are greener - that residents of more economically developed countries, as well as relatively wealthier people within countries, are more concerned about the state of the natural environment and more willing to pay to protect.” Nevertheless, the research by Fairbrother (2013, p. 910) concludes that “environmental concern is generally higher in poorer countries, and there is no relationship over time between economic development and people’s willingness to pay for environmental protection. Within countries, richer people are slightly more concerned about the environment, but only on some dimensions and not others.” There are actually different views on relationships among economic growth, values, inequality and environment (e.g., Inglehart, 1995; Brechin, 1999; Gelissen, 2007; Dunlap and York, 2008; Franzen and Meyer, 2010; Givens and Jorgenson, 2011).

As pointed out by Fullerton and Kim (2008), existing research has proposed a number of different models for analyzing different questions not in an integrated way. This study deals with growth and environmental change within an integrated framework of the Walrasian general equilibrium and neoclassical growth theories. The two theories have played the key role in the development of formal economic theories in modern times. The Walrasian general equilibrium theory was initially developed by Walras. Its sophistication in mathematical sense was done mainly the 1950s by Arrow, Debreu and others (e.g., Walras, 1874; Arrow and Debreu, 1954; Gale, 1955; Nikaido, 1956, 1968; Debreu, 1959; McKenzie, 1959; Arrow and Hahn, 1971; Arrow, 1974; Mas-Colell *et al.*, 1995; and Impicciatore *et al.*, 2012). The theory is essential for explaining equilibrium of pure economic exchanges with heterogeneous supplies and households. Nevertheless, few formal models in economics are successful in extending the theory to include endogenous wealth, not to mention endogenous environment. Walras failed in developing a general equilibrium theory with endogenous saving and capital accumulation (e.g.,). Over years many economists attempted to further develop Walras' capital accumulation within Walras' framework (e.g., Morishima, 1964, 1977; Diewert, 1977; Eatwell, 1987; Dana *et al.*, 1989; and Montesano, 2008). Nevertheless, no study succeeds in solving the common problem of lacking proper microeconomic foundation for wealth accumulation. On the other hand, the neoclassical growth theory directly models endogenous wealth accumulation with microeconomic foundation (e.g., Ramsey model). We will integrate the neoclassical growth theory with the Walrasian general equilibrium theory for studying dynamic interactions among growth, wealth and income distribution, and economic structures. It should be noted that some attempts have been made to introduce neoclassical growth theory into the general equilibrium analysis (e.g., Jensen and Larsen, 2005). As reviewed by Shoven and Whalley (1992, p. 1), "Most contemporary applied general models are numerical analogs of traditional two-sector general equilibrium models popularized by James Meade, Harry Johnson, Arnold Harberger, and others in the 1950s and 1960s." Only a few formal dynamic models explicitly deal with distribution issues among heterogeneous households in the neoclassical growth theory (Solow, 1956; Burmeister and Dobell, 1970; and Barro and Sala-i-Martin, 1995). This paper is based on the two models proposed by Zhang (2012, 2013). Zhang (2012) introduces heterogeneous households into the neoclassical growth theory without environmental dynamics, while Zhang (2013) deals with environment and growth with a homogeneous population. The two models are based on an alternative approach to household behavior proposed by Zhang (1993). The main contribution of this paper is to synthesize the economic mechanisms in Zhang's previous two models in a comprehensive framework. By doing this, this paper finds some interactions between environment and economic growth with endogenous inequality. The paper is organized as follows. Section 2 develops the growth model of wealth and income distribution with endogenous wealth and environment accumulation. Section 3 examines dynamic properties of the model and simulates the model with three groups. Section 4 carries out comparative dynamic analysis with regard to the influence of ethnic externalities, the populations, the change in the total productivities, and the propensity to save. Section 5 concludes the study.

## 2. THE BASIC MODEL

This study introduce environment into a three-sector neoclassical growth model with heterogeneous households. The production side consists capital goods and consumer goods sectors. The government financially supports an environmental sector. The capital goods and consumer goods sectors are the same as in the Uzawa two sector model (Uzawa, 1961). Here, services belong to the consumer goods sector. Most aspects of the production sectors are neoclassical (see Burmeister and Dobell 1970; Azariadis, 1993; and Barro and Sala-i-Martin, 1995). Capital goods are be used as inputs in the three sectors. Capital depreciates at a constant exponential rate $\delta_k$, being independent of the manner of use. Saving is undertaken only by households. Households own assets of the economy and distribute their incomes to consume and save. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. All earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill and capital ownership. The population is classified into *J* groups. Each group has a fixed population, $N_i(t)$, ($j = 1, ..., J$). It should be remarked that in the Walrasian general equilibrium theory, $\overline{N}_j = 1$ for all $j$. Let prices be measured in terms of capital goods and the price of capital goods be unity. We denote the wage rate of worker of household $j$ and rate of interest by $w_j(t)$ and $F_i(t)$ respectively. Let $p(t)$ denote the price of consumer goods. The total capital stock $K(t)$ is allocated among the three sectors. The total population $\overline{N}$ and total qualified labor supply $N$ are

$$\overline{N} = \sum_{j=1}^{J} \overline{N}_j, \quad N = \sum_{j=1}^{J} h_j \overline{N}_j, \tag{1}$$

in which $h_j$ is the human capital of group $j$. As we consider the population of each group and human capital constant, the total population and total labor supply are constant.

### 2.1. The Capital Goods Sector

We use subscript index, *i*, *s* and *e* to stand for capital goods and consumer goods sectors, respectively. We use $N_m(t)$ and $K_m(t)$ to stand for the labor force and capital stocks employed by sector *m*. We assume that production of capital goods is to combine labor force, $N_i(t)$, and physical capital, $K_i(t)$. We use the conventional production function to describe a relationship between inputs and output, except that environmental quality affects productivity. Let $F_i(t)$ stand for the output level of the capital goods sector at time *t*. The production function is specified as follows

$$F_i(t) = A_i \Gamma_i(E) K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \;\; A_i, \alpha_i, \beta_i > 0, \;\; \alpha_i + \beta_i = 1, \tag{2}$$

where $A_i$, $\alpha_i$ and $\beta_i$ are positive parameters. Here, $\Gamma_i(E)$ is a function of the environmental quality measured by the level of pollution, $E(t)$. It is reasonable to assume that productivity is negatively related to the pollution level, i.e., $d\Gamma_i / dE \le 0$.

Markets are competitive; thus labor and capital earn their marginal products. The rate of interest, $r(t)$, and wage rate, $w(t)$, are determined by markets. We use $\tau_i$ to stand for the fixed tax rate on the capital goods sector. The marginal conditions are given by

$$r(t) + \delta_k = \frac{\alpha_i \bar{\tau}_i F_i(t)}{K_i(t)}, \;\; w(t) = \frac{\beta_i \bar{\tau}_i F_i(t)}{N_i(t)}, \tag{3}$$

where $\bar{\tau}_i \equiv 1 - \tau_i$, $0 < \tau_i < 1$. The wage rate of group *j* is given as follows

$$w_j(t) = h_j w(t). \tag{4}$$

### 2.2. The Consumer Goods Sector

The production function of the consumer goods sector is

$$F_s(t) = A_s \Gamma_s(E) K_s^{\alpha_s}(t) N_s^{\beta_s}(t), \;\; \alpha_s + \beta_s = 1, \;\; \alpha_s, \beta_s > 0, \tag{5}$$

where $A_s$, $\alpha_s$ and $\beta_s$ are the technological parameters of the consumer goods sector and $\Gamma_i(E)$ is a function of the environmental quality. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_s \bar{\tau}_s p(t) F_s(t)}{K_s(t)}, \;\; w(t) = \frac{\beta_s \bar{\tau}_s p(t) F_s(t)}{N_s(t)}. \tag{6}$$

where $\tau_s$ is the fixed tax rate on the consumer goods sector and $\bar{\tau}_s \equiv 1 - \tau_s$, $0 < \tau_s < 1$.

### 2.3. Environmental Change

There are close interactions among economic growth, consumption and environmental changes (For instance, John and Pecchenino, 1994; Prieur, 2009). As a society accumulates more capital and makes progresses in technology, more resources may be used to protect environment. Tradeoffs between consumption and pollution have been extensively analyzed since the publication of the seminal papers by Plourde (1972) and Forster (1973). To properly model dynamics of environment, we need to know how different factors may affect environmental change (e.g., Lamla, 2009). As Gassebner *et al.* (2011, p. 588) observed, "Despite empirical research investigating the impact of various economic, political, and demographic factors on pollution, however, there is no consensus over which of these factors actually matter." This study views environmental change as an endogenous variable in the sense that it is not only determined by behavior of firms, households and the government, but also affects behavior of firms, households and the government. Economic growth often implies worsened environmental conditions. Growth also implies a higher material standard of living which will, through the demand for a better environment induces changes in the structure of the economy to improve environment. We now describe dynamics of the stock of pollutants, $E(t)$. We assume that pollutants are created both by production and consumption. We specify the dynamics of the stock of pollutants as follows

$$\dot{E}(t) = \theta_i F_i(t) + \theta_s F_s(t) + \sum_{j=1}^{J} \theta_j C_j(t) - Q_e(t) - \theta_0 E(t), \tag{7}$$

in which $\theta_i, \theta_x, \theta_j$ and $\theta_0$ are positive parameters and

$$Q_e(t) = A_e \Gamma_e(E) K_e^{\alpha_{e0}}(t) N_e^{\beta_{e0}}(t), \ \ A_e, \alpha_{e0}, \beta_{e0} > 0, \tag{8}$$

where $A_e$, $\alpha_e$ and $\beta_e$ are positive parameters, and $\Gamma_e(E)(\geq 0)$ is a function of $E$. The term $\theta_i F_i$ means that pollutants that are emitted during production processes are linearly positively proportional to the output level (for instance, Gutiérrez, 2008). The parameter, $\theta_j$, means that in consuming one unit of the good the quantity $\theta_j$ is left as waste. The parameters $\theta_j$ depend on the technology and environmental sense of consumers. The parameter $\theta_0$ is called the rate of natural purification. The term $\theta_0 E$ measures the rate that the nature purifies environment. The term, $K_e^{\alpha_e} N_e^{\beta_e}$, in $Q_e$ means that the purification rate of environment is positively related to capital and labor inputs. The function, $\Gamma_e(E)$, implies that the purification efficiency is dependent on the stock of pollutants. It is not easy to generally specify how the purification efficiency is related to the scale of pollutants. For simplicity, we specify $\Gamma_e$ as follows $\Gamma_e(E) = \theta_e E^{\upsilon}$, where $\theta_e > 0$ and $\upsilon > 0$ are parameters. As far as economic production, capital accumulation and environmental dynamic are concerned, our model is similar to the dynamic model by Dinda (2005) in many aspects. Like in Dinda's model, we allow capital allocation between commodity production and pollution abatement; but different from Dinda's model in which labor is omitted in the economy and neglect possible pollution due to consumption, we allow labor allocation between commodity production and pollution abatement and explicitly treat consumption as a source of pollution. It is important to take the pollution due to consumption into consideration when dealing with relations between environment and growth. Moreover, our modelling framework with heterogeneous households is unique.

### 2.4. Consumer Behaviors

In this study, we use an alternative approach to modeling behavior of households proposed by Zhang (1993). In addition to the environmental taxation on firms (outputs), we also take account of taxation on wealth income, consumption and wage income. There are models with environmental tax incidence (see, for instance, Rapanos, 1992, 1995). Our approach differs from the traditional approaches also with regard to how the environmental taxation affects behavior of households. Consumers make decisions on choice of consumption levels of goods as well as on how much to save. Let $\bar{k}_j(t)$ stand for per capita wealth of group *j*. We have $\bar{k}_j(t) = \bar{K}_j(t)/\bar{N}_j$, where $\bar{K}_j(t)$ is the total wealth held by group *j*. We use $\tau_{kj}$ and $\tau_{wj}$ to respectively stand for the tax rates on the wealth income and wage income. Per capita current income from the interest payment $r(t)\bar{k}_j(t)$, and the wage payment $w_j(t)$, is

$$y_j(t) = \bar{\tau}_{kj}\, r(t)\bar{k}_j(t) + \bar{\tau}_{wj}\, w_j(t),$$

where $\bar{\tau}_{kj} \equiv 1 - \tau_{kj}$ and $\bar{\tau}_{wj} \equiv 1 - \tau_{wj}$. The per capita disposable income is the sum of the current disposable income and the value of wealth. That is

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \tag{9}$$

The disposable income is used for saving and consumption. It should be noted that the value, $\bar{k}_j(t)$, (i.e., $p(t)\bar{k}_j(t)$ with $p(t) = 1$), in the above equation is a flow variable. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, we consider $\bar{k}_j(t)$ as the amount of the income that the consumer obtains at time *t* by selling all of his wealth. Hence, at time *t* the consumer has the total amount of income equaling $\hat{y}_j(t)$ to distribute between saving and consumption.

The disposable income is used for saving and consumption. At each point of time, a consumer would distribute the total available budget between saving $s_j(t)$ and consumption $c_j(t)$. The budget constraint is given by

$$(1+\tau_{cj})p(t)c_j(t)+s_j(t)=\hat{y}_j(t), \tag{10}$$

where $\tau_{cj}$ is the tax rate on consumption. It should be noted that there are different taxes on households as well as producers (e.g., Bovenberg and Smulders, 1995; Bovenberg *et al.*, 2008). Our approach takes account of different exogenous tax rates. Another important issue is how to endogenously determine the tax rates. In our model, at each point of time, consumers have two variables, $s_j(t)$ and $c_j(t)$, to decide. For simplicity of analysis, we specify the utility function as follows

$$U_j(t)=\Gamma_j(E(t))c_j^{\xi_{0j}}(t)s_j^{\lambda_{0j}}(t),\ \ \xi_{0j},\lambda_{0j}>0, \tag{11}$$

where $\Gamma_j(E)$ is a function related to the environment, $\xi_{0j}$ is the propensity to consume and $\lambda_{0j}$ the propensity to own wealth. It should be noted that this study does not explicitly take account of consumers' awareness of environment. For instance, consumers may prefer to environment-friendly goods when their living conditions are changed. With regard to how much money the economic agent should spend on environmental improvement, Selden and Song (1995) hold that at a lower level of pollution, the representative agent does not care much about environment and spends his resource on consumption; however, as the environment becomes worse and income becomes higher, more capital will be used for environmental improvement. We may take account of changes in consumers' behavior, for instance, by assuming that the representative consumer spends a proportion of the disposable income on environment or the tax rate on the consumer's consumption is explicitly related to income and consumption level.

For the representative consumer, wage rate $w_j(t)$ and rate of interest $r(t)$ are given in markets and wealth $k_j(t)$ is predetermined before decision. Maximizing $U_j(t)$ subject to the budget constraint (9) yields

$$p(t)c_j(t)=\xi_j\,\hat{y}_j(t),\ \ s_j(t)=\lambda_j\,\hat{y}_j(t), \tag{12}$$

where

$$\xi_j \equiv \frac{\rho_j \xi_{0j}}{1+\tau_{cj}}, \ \lambda_j \equiv \rho_j \lambda_{0j}, \ \rho_j \equiv \frac{1}{\xi_{0j}+\lambda_{0j}} .$$

In Balcao (2001) and Nakada (2004), it is assumed that utility depends negatively on pollution, which is a side product of the production process. As reviewed by Munro (2009, p. 43), "environmental economics has been slow to incorporate the full nature of the household into its analytical structures. … [A]n accurate understanding household behavior is vital for environmental economics."

We now find dynamics of capital accumulation. According to the definition of $s_j(t)$ the change in the household's wealth is given by

$$\dot{\bar{k}}_j(t)=s_j(t)-\bar{k}_j(t). \tag{13}$$

The equation simply states that the change in wealth is equal to saving minus dissaving.

### 2.5. The Capital and Labor Employed by the Environment Sector

We now determine how the government determines the number of labor force and the level of capital employed for purifying pollution. We assume that all the tax incomes are spent on environment. The government's tax incomes consist of the tax incomes on the production sector, consumption, wage income and wealth income. Hence, the government's income is

$$Y_e(t)=\tau_i F_i(t)+\tau_s F_s(t)+\sum_{j=1}^{J}\tau_{cj}c_{sj}(t)\bar{N}_j+\sum_{j=1}^{J}\tau_{wj}w_j(t)\bar{N}_j+\sum_{j=1}^{J}\tau_{kj}r(t)\bar{k}_j(t)\bar{N}_j. \tag{14}$$

Ono (2003) introduces tax on the producer and uses the tax income for environmental improvement in the traditional neoclassical growth theory. For simplicity, we assume that the government's income is used up only for the environmental purpose. As there are only two input factors in the environmental sector, the government budget is given by

$$(r(t)+\delta_k)K_e(t)+w(t)N_e(t)=Y_e(t). \tag{15}$$

We need an economic mechanism to analyze how the government distributes the tax income. We assume that the government will employ the labor force and capital stocks for purifying environment in such a way that the purification rate achieves its maximum under the given budget constraint. The government's optimal problem is given by

$$\text{Max}\, Q_e(t) \quad \text{s.t.:} \quad (r(t)+\delta_k)K_e(t)+w(t)N_e(t)=Y_e(t) .$$

The optimal solution is given by

$$(r(t)+\delta_k)K_e(t)=\alpha_e Y_e(t), \;\; w(t)N_e(t)=\beta_e Y_e(t), \tag{16}$$

where

$$\alpha_e \equiv \frac{\alpha_{e0}}{\alpha_{e0}+\beta_{e0}}, \;\; \beta_e \equiv \frac{\beta_{e0}}{\alpha_{e0}+\beta_{e0}} .$$

### 2.6. Demand and Supply

The demand and supply equilibrium for the consumer goods sector is

$$\sum_{j=1}^{J} c_j(t)\bar{N}_j = F_s(t). \tag{17}$$

As output of the capital goods sector is equal to the depreciation of capital stock and the net savings, we have

$$S(t)-K(t)+\delta_k K(t)=F_i(t), \tag{18}$$

where

$$S(t) \equiv \sum_{j=1}^{J} s_j(t)\bar{N}_j, \;\; K(t)=\sum_{j=1}^{J} \bar{k}_j(t)\bar{N}_j .$$

Let $K(t)$ stand for respectively the labor supply and total capital stock. The labor force is allocated among the three sectors. As full employment of labor and capital is assumed, we have

$$K_i(t)+K_s(t)+K_e(t)=K(t), \;\; N_i(t)+N_s(t)+N_e(t)=N . \tag{19}$$

We completed the model. Irrespective of the obvious strict assumptions in our model, from a structural point of view the model is quite general in the sense that some well-known models in economics can be considered as its special cases. For instance, if the population is homogeneous and environment is constant, our model is structurally similar to the neoclassical growth model by Solow (1956) and Uzawa (1961). It is structurally similar to the Walrasain model if the wealth is also fixed. The model is built on the basis of the neoclassical growth model with endogenous environment. We introduced endogenous environment into an integrated Walrasian general-equilibrium and neoclassical-growth theory.

## 3. THE DYNAMICS AND ITS PROPERTIES

The dynamic system consists of any finite number of (types of) households. The system is nonlinear and highly dimensional. It is quite difficult to get explicitly analytical properties of such a nonlinear dynamic system. Nevertheless, we can rely on computer simulation to follow the motion of the dynamic system. We now provide a computational procedure for calculating all the variables at any point of time. First, we introduce a new variable $z(t)$

$$z(t) \equiv \frac{r(t)+\delta_k}{w(t)}.$$

**Lemma**

The motion of the economic system is determined by $J + 1$ differential equations with $E(t)$, $z(t)$ and $\{\bar{k}_j(t)\}$, where $\{\bar{k}_j(t)\} \equiv (\bar{k}_2(t), \dots, \bar{k}_J(t))$ as the variables

$$\begin{aligned} &\dot{E}(t) = \varphi_0(E(t), z(t), \{\bar{k}_j(t)\}), \\ &\dot{z}(t) = \varphi_1(E(t), z(t), \{\bar{k}_j(t)\}), \\ &\dot{\bar{k}}_j(t) = \varphi_j(E(t), z(t), \{\bar{k}_j(t)\}), \ j = 2, \dots, j, \end{aligned} \tag{20}$$

in which $\varphi_j(E(t), z(t), \{\bar{k}_j(t)\})$ are unique functions of $E(t)$, $z(t)$ and $\{\bar{k}_j(t)\}$, defined in the appendix. At any point of time the other variables are unique functions of $E(t)$, $z(t)$ and $\{\bar{k}_j(t)\}$ by the following procedure:

$r(t)$ and $w_j(t)$ by (A2) $\rightarrow \bar{k}_1(t)$ by (A19) → $N_e(t)$ by (A17) → $N_i(t)$ by (A9) → $N_s(t)$ by (A6) → $K_e(t)$, $K_s(t)$ and $K_i(t)$ by (A1) → $w(t) = w_1(t)/h_1(t)$ → $\hat{y}_j(t)$ by (A4) → $p(t)$ by (A3) → $F_i(t)$, $F_i(t)$ and $F_s(t)$ by the definitions → $c_j(t)$ and $s_j(t)$ by (12) $\rightarrow K(t) = K_i(t) + K_s(t) + K_e(t)$ → $U_j(t)$ by the definition.

The lemma provides a computational procedure for illustrating the motion of the economic system with any number of types of households. It is well known that calibration of general equilibrium involves solving high-dimensional nonlinear equations. With regard to the Arrow-Debreu concept of general equilibrium the final stage of analysis is to find a price vector at which excess demand is zero (Judd, 1998). There are numerical approaches for calculating equilibria (e.g., Scarf, 1967; Scarf and Hansen, 1973). We can apply these traditional methods to find how the prices and other variables are related to the variables in the differential equations. As it is difficult to interpret the analytical results, to study properties of the system we simulate the model for a 3-group economy. We specify

$$\Gamma_j(E) = E^{-b_j}(t), \; b_j \ge 0, \; j = 1, 2, 3.$$

The parameter values are specified as follows

$$\tau_i = 0.02, \; \tau_s = 0.02, \; A_e = 1, \; \alpha_i = 0.38, \; \alpha_s = 0.3, \; \alpha_{e0} = 0.4, \; \beta_{e0} = 0.2, \; \delta_k = 0.05,$$

$$\begin{pmatrix} \bar{N}_1 \\ \bar{N}_2 \\ \bar{N}_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 12 \end{pmatrix}, \; \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}, \; \begin{pmatrix} \lambda_{10} \\ \lambda_{20} \\ \lambda_{30} \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.45 \\ 0.35 \end{pmatrix},$$

$$\begin{pmatrix} \xi_{10} \\ \xi_{20} \\ \xi_{30} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix}, \; \begin{pmatrix} A_i \\ A_s \\ A_e \end{pmatrix} = \begin{pmatrix} 1.3 \\ 1 \\ 1 \end{pmatrix}, \; \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.04 \\ 0.05 \end{pmatrix},$$

$$\begin{pmatrix} \tau_{c1} \\ \tau_{c2} \\ \tau_{c3} \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.01 \\ 0.01 \end{pmatrix}, \; \begin{pmatrix} \tau_{w1} \\ \tau_{w1} \\ \tau_{w1} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix}, \; \begin{pmatrix} \tau_{k1} \\ \tau_{k2} \\ \tau_{k3} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix}, \tag{21}$$

$$\begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} 0.03 \\ 0.04 \\ 0.06 \end{pmatrix}, \; \begin{pmatrix} \theta_{\mathrm{i}} \\ \theta_{\mathrm{s}} \\ \theta_0 \end{pmatrix} = \begin{pmatrix} 0.03 \\ 0.1 \\ 0.2 \end{pmatrix}, \; \begin{pmatrix} b_{\mathrm{i}} \\ b_{\mathrm{s}} \\ b_{\mathrm{e}} \end{pmatrix} = \begin{pmatrix} 0.04 \\ 0.03 \\ 0.04 \end{pmatrix}.$$

The tax rates on the two production sectors are 2 percent. The population of group 3 is largest, while the population of group 2 is the next. The capital goods, consumer goods and environmental sectors' total factor productivities are respectively 1.3, 1 and 1. We specify the values of the parameters, $\alpha_j$, in the Cobb-Douglas productions for the capital goods and consumer goods sectors approximately equal to 0.3 (for instance, Miles and Scott, 2005; Abel *et al.*, 2007). The depreciation rate of physical capital is specified at 0.05. Group 1 has the highest propensity to save and group 3 has the lowest propensity to save. Group 1 also has the highest level of human capital and group 3 has the lowest level of human capital. We require the tax rate on consumption level of any group to be one percent. The tax rates on wage income and wealth income are equally set at 2 percent for any group. In order to simulate the model, we need to know the state of the dynamic system at some point in time. As we already have the computational procedure to follow the model of the economy, we can choose any initial state. In the rest of the paper we choose the following initial conditions

$$z(0)=0.15,\ E(0)=25,\ \bar{k}_2(0)=6,\ \bar{k}_3(0)=3.5.$$

The motion of the variables is plotted in Figure 1. In Figure 1, the national income is

$$Y(t)=F_i(t)+p(t)F_s(t)\ .$$

The output level of the capital goods sector falls overtime, while the output level of the consumer goods sector rises. The price of consumer goods is increased slightly. The net impact on the national income is that the national income increased. We see that environment is improved in association with the rise in the national income. The total capital stock and capital stock employed by the consumer goods sector are increased, while the capital input of the capital goods sector is decreased. From the figure we see that as the environment is improved, different groups experience different changes in terms of wage, consumption, and wealth. In association with the improvement in environment, the rich and the middle experience rises in wage, consumption, and wealth; but the poor suffers from fallings in utility, wage and wealth, even though the poor's consumption is increased. This result also implies the necessity of introducing heterogeneous households into the dynamic analysis of environmental change. For instance, in a study by Grossman and Krueger (1995), they pay attention to the relationship per capita and various environmental indicators - urban air pollution, the state of the oxygen regime, in river basins, and fecal contamination of river basins, and contamination of river basins by heavy metals. Grossman and Krueger (1995, p. 353) concludes that they find no evidence that "environmental quality deteriorates steadily with economic growth." Our simulation indicates that although the conclusion made by Crossman and Krueger holds for the national economy, but is invalid for certain group. This also hints another aspect of complexity of environmental management. An environmental policy may bring about different benefits to different agents in the economic system.

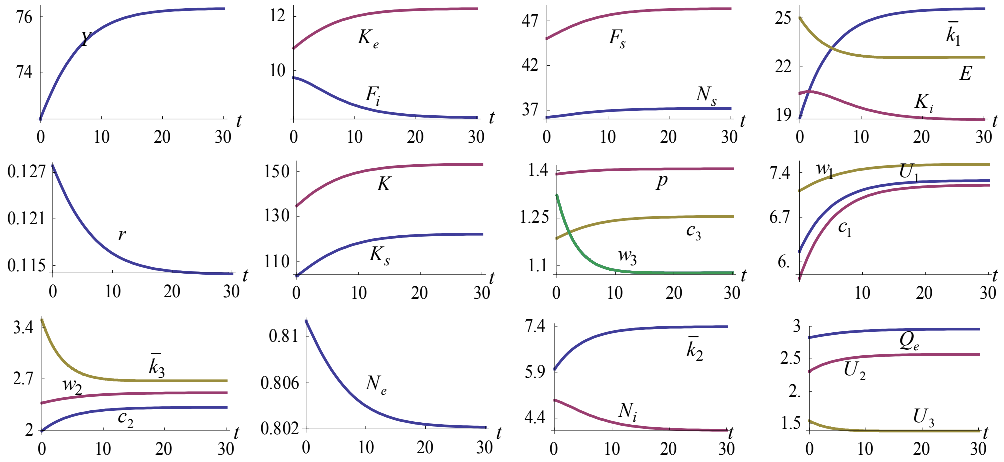

**Figure 1.** The Motion of the Economic System

It is straightforward to confirm that all the variables become stationary in the long term. This implies the existence of an equilibrium point. The simulation confirms that the system has a unique equilibrium. We list the equilibrium values in (22)

$$\begin{gathered}K=153.2,\ E=22.61,\ Y=76.3,\ Y_e=3.02,\ r=0.12,\ p=1.41,\\ w_1=7.53,\ w_2=2.51,\ w_3=1.25,\ F_i=8.25,\ F_s=48.4,\\ Q_e=2.96,\ N_i=3.99,\ N_s=37.2,\ N_e=0.8,\ K_i=18.75,\\ K_s=122.1,\ K_e=12.3,\ \bar{k}_1=25.58,\ \bar{k}_2=7.39,\ \bar{k}_3=2.67,\\ c_1=7.2,\ c_2=2.31,\ c_3=1.08,\ U_1=7.28,\ U_2=2.57,\ U_3=1.39.\end{gathered} \tag{22}$$

It is straightforward to calculate the six eigenvalues as follows

$$\{-0.28,\ -0.22,\ -0.21 \pm 0.03i\}.$$

The eigenvalues are real and negative. The unique equilibrium is locally stable. This guarantees the validity of exercising comparative dynamic analysis.

## 4. COMPARATIVE DYNAMIC ANALYSIS

We simulated the motion of the national economy under (21). We now study how the economic system reacts to changes, for instance, in tax rates and preferences. As the lemma gives a computational procedure to calibrate the motion of all the variables, we can easily conduct analysis on effects of change in any parameter on transitory processes as well stationary states of all the variables. In the rest of this study we use $\bar{\Delta} x_j(t)$ to stand for the change rate of the variable, $x_j(t)$, in percentage due to changes in the parameter value.

### 4.1. The Government Increasing the Environmental Tax Rate on the Capital Goods Sector

We first study the case when the environmental tax rate on the capital goods sector is increased as follows: $\tau_i : 0.02 \Rightarrow 0.05$. The simulation result is plotted in Figure 2. The firms in the industry now pay 5 percent tax rate on the output. The output level of the capital goods sector is reduced over time. The falling rate is initially 14 percent and gradually achieves its equilibrium level about 6 percent. As the government has more money to spend on improvement, the two inputs and output level of the environment sector are increased. The capital goods sector employ less labor and capital inputs in association with increased taxation. The total capital stock is reduced as per household's wealth is reduced. The output level of the consumer goods sector rises initially but then falls. The price of consumer goods falls over time. The rate of interest falls initially but rises in the long term. As the demand for capital falls and the total capital reduction is not large, the rate is falling. Nevertheless, in the long term the total capital is largely reduced, the rate of interest is increasing. The consumption levels of consumer goods are increased initially in association with falling in the price. The wage rates are reduced as the tax rate is increased. Nevertheless, as the households have les disposable incomes, their consumption of consumer goods fall in the long term.

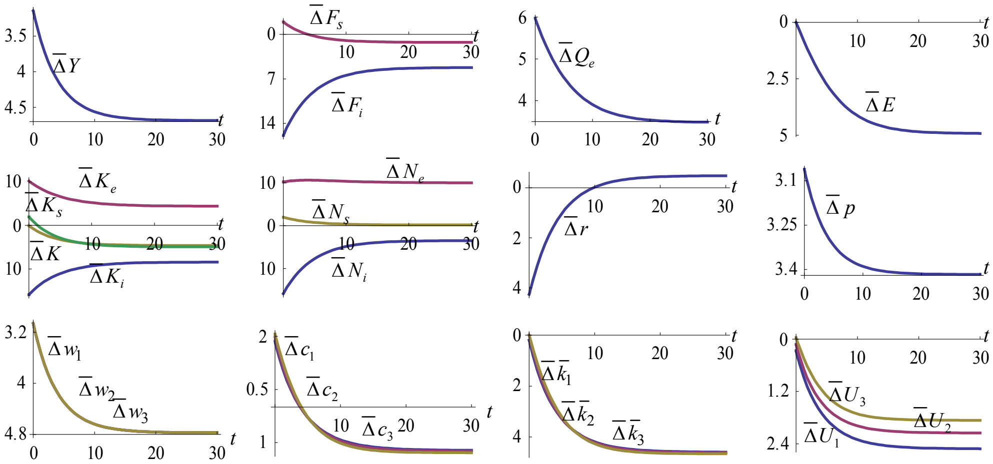

**Figure 2.** A Rise in the Tax Rate on the Capital Goods Sector

The environment is improved. Although the improved environment should enhance the utility levels of the households, the utility levels of the three groups actually fall because the wealth and consumption levels of all the three groups are reduced in the long term. The national income falls as its three components all fall in the long term. We see that a rise in the tax rate on the capital goods sector will reduce the utility levels and national income, even though the environmental quality is enhanced.

### 4.2. The Tax Rate on the Rich's Wealth Income Being Increased for Environmental Protection

We now study the impact of the following rise in the tax rate on the rich's wealth income: $\tau_{k1}: 0.02 \Rightarrow 0.04$. The simulation result is plotted in Figure 3. As the tax rate is increased, the national wealth rises slightly initially but soon falls over time. The initial rise is partly because the rich's wealth is increased initially. The rise in the tax rate on wealth tends to directly discourage the rich to hold wealth. As the rate of interest is increased and the price of consumer goods is slightly reduced, the net result on the rich's wealth is to increase the rich's wealth. Nevertheless, this tendency does not last long as the impact of taxation becomes dominant. The rich's wealth begins to fall soon after its short rising period. The government has more resources on protecting environment. The output level and two inputs of the environmental sector are increased. More efforts in protecting environment improve the environmental quality. In association of falls in the total wealth, the national income and output levels of the two production sectors fall. The rate of interest rises, while the price of consumer goods and wage rates of the three groups all fall. The consumption and wealth levels of the middle and poor are increased, while the corresponding variables of the rich fall in the long term. The utility level of the rich is increased initially but reduced in the long term, while the utility levels of the other two groups are augmented. In summary, we see that a rise in the tax rate on the rich's wealth income betters environment and enhances the utility levels of the rest groups, but lowers the national wealth and national income, and reduces the rich's wealth, consumption and utility level. It should be noted that relations between inequality and environment have caused increasing attention in economics. Because of possible complicated interactions between these two aspects of economic systems, it is expected to see that some researchers argue for positive relations and others point out negative relations. For instance, Gassebner *et al.* (2011) conclude: "In contrast to the mostly negative association so far in the literature, we confirm more recent model predictions which state that there are indeed beneficial side-effects of income inequality on the environmental quality." Our simulation does not confirm what the paper just cited predicts. It should be noted that the narrowed gap between the rich and poor in wealth and income in our study is due to the specified used of the tax income on the rich. We will show soon that if the narrowed gap between the rich and the poor is due to the rise in the poor's human capital, then a narrowed inequality is associated with environmental deterioration. Our simulation hints on different possibilities between environment and inequality in a world with government intervention as well as various determinants of inequality.

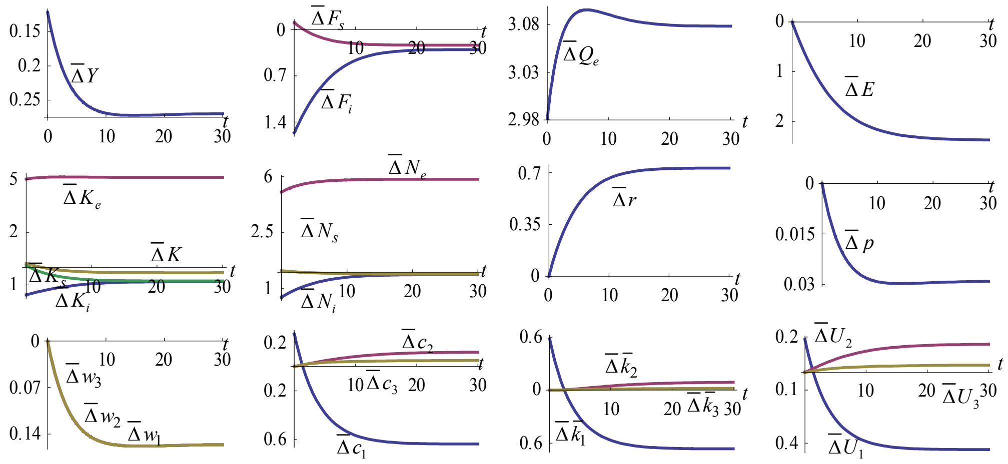

**Figure 3.** A Rise in the Tax Rate on the Rich's Wealth Income

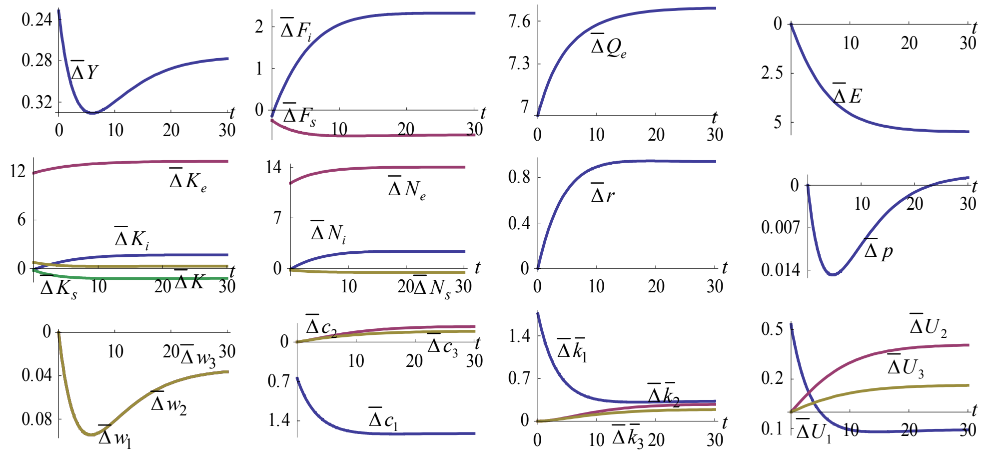

**Figure 4.** A Rise on the Tax Rate on the Rich's Consumption

### 4.3. The Government Increasing the Tax Rate on the Rich's Consumption

We examined the impact of a rise in taxation on the rich's wealth income on the national economy. We now examine what will happen to the national economy if the government increases the tax rate on the rich's consumption, instead of on the wealth income. We have the following change in the policy: $\tau_{c1} :0.01 \Rightarrow 0.03$. The simulation result is plotted in Figure 4. We see that the two changes in taxation on the rich have similar effects on the economy. But there are some differences. For instance, when the government taxes more heavily on the rich's consumption, the consumer goods sector is negatively affected but the capital goods are positively affected, while in the case of rising the tax rate on wealth income, the two sectors are negatively affected. Moreover, the rich's wealth is increased instead of being reduced.

### 4.4. The Poor's Human Capital Being Increased

Interactions among growth, inequality and human capital have caused great attention both in theoretical economic and empirical research (e.g., Easterlin 1981; Tilak, 1989; Hanushek and Kimko, 2000; Barro, 2001; Krueger and Lindahl, 2001; Castelló-Climent and Hidalgo-Cabrillana, 2012). Could *et al.* (2001) build an economic model, concluding that the primary source of inequality growth within uneducated workers is due to increasing randomness, but inequality growth within educated workers is mainly due to changes in the composition and return to ability (see also Tselios, 2008; Fleisher *et al.* 2011). Although this study treats human capital as exogenous variables, it is important for us to examine how human capital change of any group may affect the entire economy. We are now concerned with the effects of change in the poor's human capital as follows: $h_3 :1 \Rightarrow 1.5$. We plot the simulation results in Figure 5.

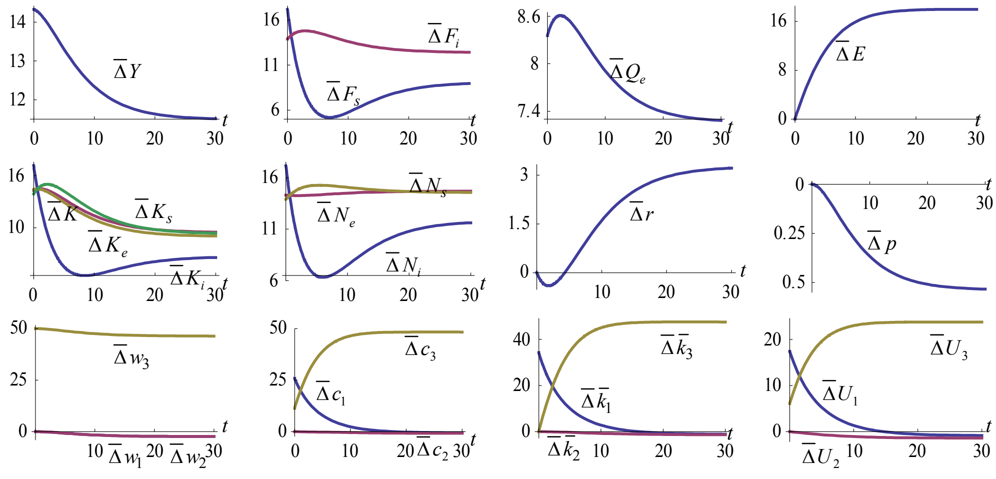

**Figure 5.** The Poor's Human Capital Being Improved

As the productivity is enhanced, the total supply of labor is increased. The output levels of the two sectors, the national income and inputs of capital and labor are all augmented. The output level and inputs of the environmental sectors are increased as the government's tax income is increased. Nevertheless, the environment deteriorates as a net result of strengthened environmental protection and increased pollutants by production and consumption. The poor's wage rate, wealth and consumption, and utility are all increased. The middle's wage rate consumption, wealth and utility levels are all lowered. The rich's wage rate falls. The rich's consumption, wealth and utility levels are augmented initially but are slightly affected in the long term.

### 4.5. The Poor's Population Being Increased

The relationship between population change and economic development is empirically ambiguous and theoretically difficult. Theoretical models with human capital predict situation-dependent interactions between population and economic growth (e.g., Ehrlich and Lui, 1997; Galor and Weil, 1999; Boucekkine *et al*., 2002; Bretschger, 2013). There are also mixed conclusions in empirical studies on the issue (e.g., Furuoka, 2009; Yao *et al*., 2013). Although this study is not concerned with endogenous population, we will study effects of changes in the population sizes. We now allow the poor's population to be increased as follows: $\overline{N}_3 : 12 \Rightarrow 13$. The results are plotted in Figure 6.

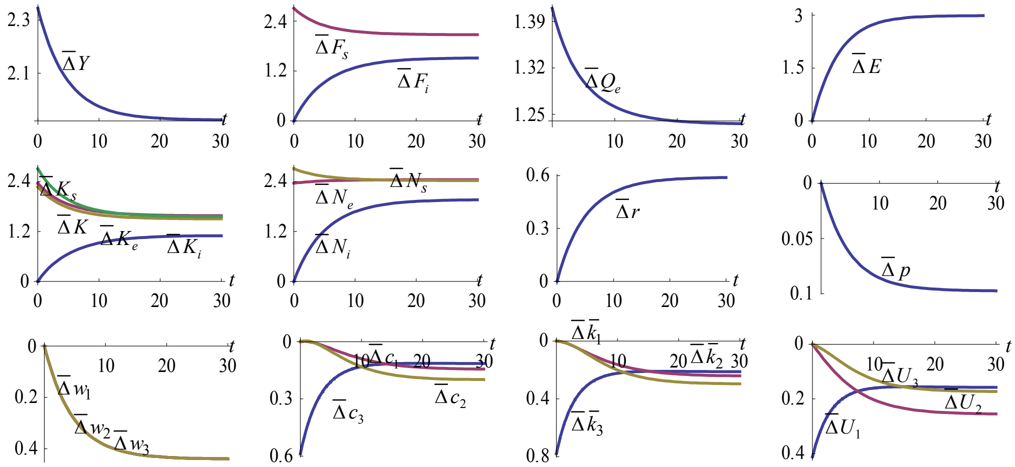

**Figure 6.** A Rise in the Poor's Population

We see that as far as the aggregate real variables are concerned, during the transitory process as well as in the long term the national income, the total capital stock, the total labor supply, the capital inputs, the labor inputs, and the output levels of the two sectors are all increased. The output of the environmental sector is increased, even though the environment deteriorates. The utility levels of all the groups are reduced. The wage rate, consumption and wealth levels and the price of consumer goods are all reduced.

### 4.6. The Poor's Propensity to Save Being Increased

In order to examine long-term effects of preference change on the national economy, one needs a general equilibrium framework with endogenous capital and environment. Economics has not yet an effective analytical framework for analyzing effects of changes in one type of households on national economic growth as well as wealth and income distribution among different households. As our analytical framework integrates the economic mechanism of the Walrasian general equilibrium theory and neoclassical growth theory, in principle we can analyze effects a change in the preference of any people on the dynamic path of the economic growth. We now allow the poor to change the propensity to save in the following way: $\lambda_{01} : 0.35 \Rightarrow 0.4$. The simulation results are given in Figure 7. The national income, the total capital stock, the total labor supply, the capital inputs, the labor inputs, and the output levels of the two sectors are all increased in the long term. The output of the environmental sector is increased, even though the environment deteriorates. The utility levels of the rich and the middle are slightly affected; the poor's utility level is enhanced. The wage rates are increased.

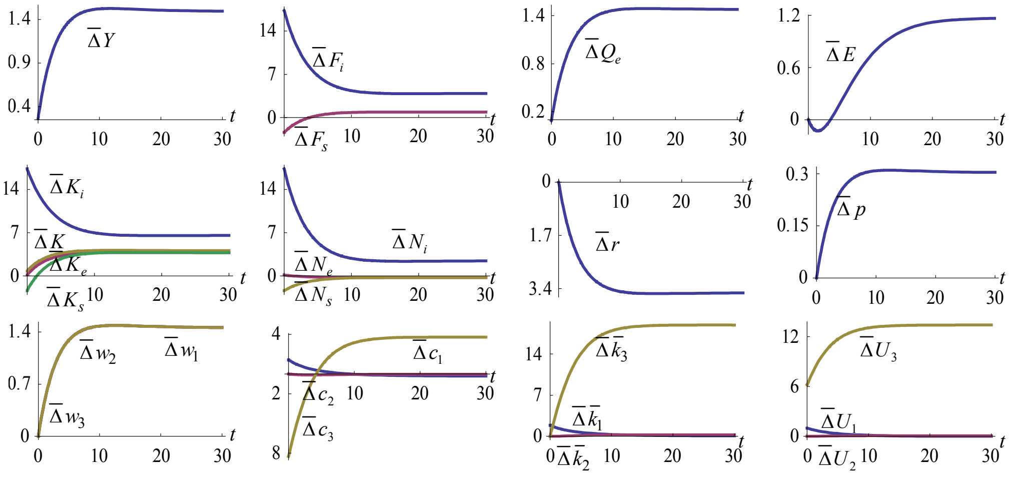

**Figure 7.** A Rise in the Poor's Propensity to Save

## 5. CONCLUDING REMARKS

This study proposed an economic growth model of heterogeneous households with economic structure and environment. The model emphasizes the role of environmental change on economic structural change and wealth and income distribution. The analytical framework is built by integrating the three important theories in economics. The three theories are the Walrasian general equilibrium theory, the neoclassical growth theory, and the neoclassical growth model with endogenous environment. The three theories are integrated by applying Zhang's approach to household behavior. The economic system consists of one capital goods sector, one consumer goods sector, one environmental sector, and any number (of types) of households. The motion is described by a set of differential equations. For illustration, we simulated the motion of the economic system with three groups. We identified the existence of a unique stable equilibrium point. We also carried out comparative dynamic analysis. We can comprehensively discuss some important issues related to growth and environmental change in a unique manner because our analytical framework contains not only the economic mechanisms for analyzing these issues, but also because we provided the computational procedure to follow the motion of the nonlinear dynamic system. The comparative analyses provide some insights into the complexity of economic growth with environment. For instance, the study by Grossman and Krueger (1995, p. 353) identifies no evidence that "environmental quality deteriorates steadily with economic growth." Our simulation indicates that although the conclusion made by Crossman and Krueger holds for the national economy, but is invalid for certain group. We showed that that as the environment is improved, different groups experience different changes in terms of wage, consumption, and wealth. In association with the improvement in environment, the rich and the middle experience rises in wage, consumption, and wealth; but the poor suffers from fallings in utility, wage and wealth, even though the poor's consumption is increased. This result also implies the necessity of introducing heterogeneous households into the dynamic analysis of environmental change. We may extend and generalize the model in different ways. For instance, as the referee points out, it is important to extend the model to a multi-country economy, to properly deal with the role of government in the whole economy, to study interactions among national governments with regard to environmental issues.

## APPENDIX

### Proving the Lemma

By (3), (6) and (16), we obtain

$$z \equiv \frac{r+\delta_k}{w} = \frac{N_m}{\bar{\beta}_m K_m}, \;\; m = i, s, e, \tag{A1}$$

where $\bar{\beta}_m \equiv \beta_m / \alpha_m$. Insert (A1) in (3)

$$r = \alpha_r \, \Gamma_i \, z^{\beta_i} - \delta_k , \;\; w_j = \alpha_j \, \Gamma_i \, z^{-\alpha_i}, \tag{A2}$$

where

$$\alpha_r = \alpha_i \, \bar{\tau}_i \, \bar{\beta}_i^{\beta_i} \, A_i , \;\; \alpha_j = \frac{\beta_i \, \bar{\tau}_i \, A_i \, h_j}{\bar{\beta}_i^{\alpha_i}}.$$

Hence, we determine the rate of interest and the wage rates as functions of $z$ and $E$. From (5) and (6), we have

$$p = \frac{\bar{\beta}_s^{\alpha_s} \, z^{\alpha_s} \, w}{\beta_s \, \bar{\tau}_s \, A_s \, \Gamma_s}, \tag{A3}$$

where $w = w_1 / h_1$. From (A2) and the definitions of $\hat{y}_j$, we have

$$\hat{y}_j = (1 + \bar{\tau}_{kj} r) \bar{k}_j + \bar{\tau}_{wj} \, w_j . \tag{A4}$$

Insert $p c_j = \xi_j \, \hat{y}_j$ in (17)

$$\sum_{j=1}^{J} \xi_j \, \bar{N}_j \, \hat{y}_j = p F_s . \tag{A5}$$

Substituting (A4) in (A5) yields

$$N_s = \sum_{j=1}^{J} g_j \, \bar{k}_j + g_0 , \tag{A6}$$

where we use $w N_s = \beta_s \, \bar{\tau}_s \, p F_s$ and

$$g_j(z, E) \equiv \left( \frac{1 + \bar{\tau}_{kj} \, r}{w} \right) \beta_s \, \bar{\tau}_s \, \xi_j \, \bar{N}_j , \;\; g_0 = \beta_s \, \bar{\tau}_s \sum_{j=1}^{J} \bar{\tau}_{wj} \, \xi_j \, \bar{N}_j \, h_j .$$

From (A1), (1) and $K = \sum_{j=1}^{J} \bar{k}_j \bar{N}_j$, we get

$$\frac{N_i}{\bar{\beta}_i} + \frac{N_s}{\bar{\beta}_s} + \frac{N_e}{\bar{\beta}_e} = z \sum_{j=1}^{J} \bar{k}_j \bar{N}_j . \tag{A7}$$

From (A6) and (A7), we solve

$$\frac{\bar{\beta}_s N_i}{\bar{\beta}_i} + \frac{\bar{\beta}_s N_e}{\bar{\beta}_e} = \bar{\beta}_s z \sum_{j=1}^{J} \bar{k}_j \bar{N}_j - \sum_{j=1}^{J} g_j \bar{k}_j - g_0 . \tag{A8}$$

Insert (A6) in $N_i + N_s + N_e = N$,

$$N_i + N_e = N - \sum_{j=1}^{J} g_j \bar{k}_j - g_0 . \tag{A9}$$

Insert (A9) in (A8)

$$\bar{\beta} N_e = \sum_{j=1}^{J} (\bar{\beta}_s z \bar{N}_j + \beta g_j) \bar{k}_j + \beta g_0 - \frac{\bar{\beta}_s}{\bar{\beta}_i} N , \tag{A10}$$

where

$$\bar{\beta} \equiv \left( \frac{1}{\bar{\beta}_e} - \frac{1}{\bar{\beta}_i} \right) \bar{\beta}_s , \quad \beta \equiv \frac{\bar{\beta}_s}{\bar{\beta}_i} - 1.$$

Substituting (A1) into (2) and (5) yields

$$F_i = \frac{A_i \Gamma_i N_i}{\bar{\beta}_i^{\alpha_i} z^{\alpha_i}} , \quad F_s = \frac{A_s \Gamma_s N_s}{\bar{\beta}_s^{\alpha_s} z^{\alpha_s}} . \tag{A11}$$

Insert (A11) in (14)

$$Y_e = \Lambda_i N_i + \Lambda_s N_s + \sum_{j=1}^{J} \tau_{cj} c_j \bar{N}_j + \sum_{j=1}^{J} \tau_{wj} w_j \bar{N}_j + \sum_{j=1}^{J} \tau_{kj} r \bar{k}_j \bar{N}_j , \tag{A12}$$

where

$$\Lambda_i(z,E) \equiv \frac{\tau_i\, A_i\, \Gamma_i}{\bar{\beta}_i^{\alpha_i}\, z^{\alpha_i}}, \;\; \Lambda_s(z,E) \equiv \frac{\tau_s\, A_s\, \Gamma_s\, N_s}{\bar{\beta}_s^{\alpha_s}\, z^{\alpha_s}}.$$

From $p\, c_j = \xi_j\, \hat{y}_j$ and (A5), we have

$$c_j = \left( \frac{1 + \bar{\tau}_{kj}\, r}{p} \right) \xi_j \bar{k}_j + \frac{\bar{\tau}_{wj}\, \xi_j\, w_j}{p}. \tag{A13}$$

Substituting (A13) into (A12) yields

$$Y_e = \Lambda_0 + \Lambda_i\, N_i + \Lambda_s\, N_s + \sum_{j=1}^{J} \Lambda_j\, \bar{k}_j \;, \tag{A14}$$

where

$$\Lambda_j(z,E) \equiv \left[ \left( \frac{1 + \bar{\tau}_{kj}\, r}{p} \right) \xi_j\, \tau_{cj} + \tau_{kj}\, r \right] \bar{N}_j, \;\; \Lambda_0(z,E) \equiv \sum_{j=1}^{J} \left( \tau_{wj} + \frac{\bar{\tau}_{wj}\, \xi_j\, \tau_{cj}}{p} \right) \bar{N}_j\, w_j .$$

Insert (A14) in $w N_e = \beta_e\, Y_e$ in (16)

$$\frac{w N_e}{\beta_e} = \Lambda_0 + \Lambda_i\, N_i + \Lambda_s\, N_s + \sum_{j=1}^{J} \Lambda_j \bar{k}_j . \tag{A15}$$

Substituting $N_i = N - N_s - N_e$ in (A15) yields

$$\left( \frac{w}{\beta_e} + \Lambda_i \right) N_e = \Lambda_0 + N\, \Lambda_i + (\Lambda_s - \Lambda_i) N_s + \sum_{j=1}^{J} \Lambda_j\, \bar{k}_j . \tag{A16}$$

Insert (A7) in (A16)

$$N_e = \Psi + \sum_{j=1}^{J} \Psi_j \bar{k}_j , \tag{A17}$$

where

$$\Psi(z,E) \equiv \left[ \Lambda_0 + N\, \Lambda_i + (\Lambda_s - \Lambda_i) g_0 \right] \left( \frac{w}{\beta_e} + \Lambda_i \right)^{-1} .$$

$$\Psi_j(z,E) \equiv \left[(\Lambda_s - \Lambda_i)g_j + \Lambda_j\right]\left(\frac{w}{\beta_e} + \Lambda_i\right)^{-1}.$$

Insert (A17) in (A10)

$$\sum_{j=1}^{J} \overline{\Psi}_j \bar{k}_j = \bar{\beta}\Psi - \beta g_0 + \frac{\bar{\beta}_s}{\bar{\beta}_i} N, \tag{A18}$$

where

$$\overline{\Psi}_j(z,E) \equiv \bar{\beta}_s z \overline{N}_j + \beta g_j - \bar{\beta}\Psi_j.$$

Solve (A20) with $\bar{k}_1$ as the variable

$$\bar{k}_1 = (z,E,\{\bar{k}_j\}) \equiv \left(\bar{\beta}\Psi - \beta g_0 + \frac{\bar{\beta}_s}{\bar{\beta}_i} N - \sum_{j=2}^{J} \overline{\Psi}_j \bar{k}_j\right)\frac{1}{\overline{\Psi}_1}, \tag{A19}$$

where $\{\bar{k}_j\} \equiv (\bar{k}_2, ..., \bar{k}_J)$. It is straightforward to confirm that all the variables can be expressed as functions of $z$, $E$ and $\{\bar{k}_j\}$ by the following procedure: $r$ and $w_j$ by (A2) → $\bar{k}_1$ by (A19) → $N_e$ by (A17) → $N_i$ by (A9) → $N_s$ by (A6) → $K_e$, $K_s$, and $K_i$ by (A1) → $w = w_1 / h_1$ → $\hat{y}_j$ by (A4) → $p$ by (A3) → $F_i$, $F_i$ and $F_s$ by the definitions → $c_j$ and $s_j$ by (12) → $K = K_i + K_s + K_e$ → $U_j$ by the definition. From this procedure, (A19), (7) and (13), we have

$$\dot{\bar{k}}_1 = \overline{\Phi}_1(z,E,\{\bar{k}_j\}) \equiv \lambda_1 \hat{y}_1 - \varphi, \tag{A20}$$

$$\dot{E} = \Phi_0(z,E,\{\bar{k}_j\}) \equiv \theta_i F_i + \theta_s F_s + \sum_{j=1}^{J} \bar{\theta}_{sj} c_j \overline{N}_j - Q_e - \theta_0 E,$$

$$\dot{\bar{k}}_j = \Phi_j(z,E,\{\bar{k}_j\}) \equiv \lambda_j \hat{y}_j - \bar{k}_j, \quad j = 2, ..., J. \tag{A21}$$

Taking derivatives of Equation (A19) with respect to $t$ and combining with (A21) implies

$$\dot{\bar{k}}_1 = \frac{\partial}{\partial z}\dot{z} + \Phi_0 \frac{\partial}{\partial E} + \sum_{j=2}^{J} \Phi_j \frac{\partial}{\partial \bar{k}_j}. \tag{A22}$$

Equaling the right-hand sizes of Equations (A20) and (A22), we get

$$\dot{z} = \Phi_1(z, E, \{\bar{k}_j\}) \equiv \left[\overline{\Phi}_1 - \Phi_0 \frac{\partial}{\partial E} - \sum_{j=2}^{J} \Phi_j \frac{\partial}{\partial \bar{k}_j}\right] \left(\frac{\partial}{\partial z}\right)^{-1}. \qquad \text{(A23)}$$

In summary, we proved the lemma.

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