BUNDLING COMPETITION BETWEEN MULTI-PRODUCT AND SINGLE-PRODUCT FIRMS

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This paper analyzes a simple model where a multi-product firm competes with single-product firms possibly with bundling strategy. Mixed bundling is theoretically known as an effective business tool even in the symmetric competition as well as for the monopoly. Contrary to the literature, this paper shows that mixed bundling is dominated by component pricing or pure bundling. The result holds regardless of the product complementarity by the multi-product firm. In addition, it is shown that linear component pricing will be utilized for low complementarity while pure bundling strategy will be chosen for high complementarity.

Keywords: Duopolies, Bundling, Bundle Discount, Complementary Goods, Nonlinear Pricing, Product Differentiation, Price Discrimination

JEL classification: D43, L13

1. INTRODUCTION

Most price theories assume that a single price is placed on each product. The market for each product arrives at the equilibrium where the unique price equalizes the demand and the supply. Regarding how sensitive the demand for a product is, we take into account other goods such as complements or substitutes. Despite the existence of complementary or substitutable goods, the assumption of one-for-one relationship between product and price had been rarely challenged.

In a real world, we often observe price discounts for multi-product purchases. If you buy more products of the same kind, you might get more discounts for an additional unit. Or if you buy several unrelated goods from the same vendor which supplies various types of products, you might be offered a discount on the bundle. In other words, the price of a bundle is placed on top of the stand-alone prices. We can theoretically devise \((2^n - 1)\) prices for all possible combinations with \(n\) single products. For example, with two products, there are three prices: two stand-alone prices and one bundle price. As the

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marketing strategies become sophisticated, multiple intra-or inter-product purchases tend to be priced differently from the simple sum of component prices. Beyond the traditional strategy of component pricing, the bundle discount is utilized to entice customers into buying multiple products from the same vendor.

Economics literature had paid scant attention to bundling strategy, a kind of nonlinear pricing, until Stigler (1963) studied the block booking in the US motion picture industry. It is suggested that a monopolist is willing to offer a bundle if consumer valuation on the products are heterogeneous. Following Stigler (1963), early literature of bundling mainly centers on the monopoly theory. Adams and Yellen (1976) illustrate prominent examples explaining why pure or mixed bundling benefits the monopolist more than component pricing. Negative correlation of consumer valuation on product space is a key factor that invites a bundling. McAfee, McMillan, and Whinston (1989) extend Adams and Yellen (1976) in continuous distribution of valuations. They derive a condition in which mixed bundling dominates other pricing strategies. Armstrong (1996) and Rochet and Choné (1998) study the bundling strategy in broader perspective of multidimensional nonlinear pricing. In the monopolist approach, bundling is an effective tool that sorts consumers into different groups for price discrimination. Offering a discounted bundle, the firm is able to distinguish the customers who prefer all the products from those who value only specific subset of the products. Thus a monopolistic multi-product firm would utilize the bundle offers to obtain the information of consumer preference.

In practice, a multi-product firm usually competes with other suppliers. The competitors might contend in each product market, or they might contend employing a similar bundle offering. Compared to the monopoly bundling literature, there are few theoretical papers pertaining to competitive bundling. Spulber (1979) shows that there exists a unique non-cooperative equilibrium of price-discriminating firms. Yet nonlinear pricing with multi-product feature was not considered earlier than Armstrong and Vickers (2001), and Rochet and Stole (2002) investigated the non-linear pricing strategy of intra-product bundle under competition. Especially, Yin (2004) focuses on two-part tariff competition.

Armstrong and Vickers (2009) build a pioneering “two-stop shopping” model for a duopoly bundling. They assume there are two symmetric firms supplying two products each. Consumers who buy both products from the same firm pay only the bundle price. Others who buy the products, from different sellers so called “mix-and-match,” bear an additional “shopping cost.” One of the main results is that two multi-product firms would employ the mixed bundling strategy in equilibrium. Armstrong and Vickers (2009) theoretically explore the use of mixed bundling in a competitive model and conclude that the mixed bundling constitutes an equilibrium even in the symmetric duopoly as well as for the monopoly.

In the literature, empirical research on competitive bundling is rarer than the theoretical one. Gandal, Markovich, and Riordan (2005) analyze the importance of bundling strategy in ‘PC Office Suite’ market. PC office software market from 1991 to
1998 had experienced the changes in marketing strategy. Release of Microsoft Office Suite in 1990 gave rise to the new generation of office software product bundles. Microsoft’s use of mixed bundling strategy was going along with its achievement of the dominant PC office suite producer. The dominance persisted even though other competing firms later offered their own office suite cooperatively. The high evaluation of the product integration is also suspected to reinforce the successful use of mixed bundling.

This paper is motivated mainly by Gandal, Markovich, and Riordan (2005) and Armstrong and Vickers (2009). A practical use of mixed bundling needs to be examined under competitive surroundings. Armstong and Vickers (2009) postulate that the mixed bundling forms an equilibrium in symmetric competition. However asymmetric competition between multi-product firm and single-product firms, introduced by Gandal, Markovich, and Riordan (2005), has remained open, and this paper explores whether a multi-product firm has an incentive to utilize the mixed bundling while competing against single-product firms.

The validity of using mixed bundling will be inspected throughout this paper. Section 2 introduces a basic model containing features, such as ‘product complementarity’ or ‘compatibility’. Section 3 indicates the competitive equilibria under different modes of pricing strategies: linear component pricing, pure bundling, and mixed bundling. At the end of analysis, I compare the equilibrium outcomes and conclude that the mixed bundling is dominated either by the linear pricing or the pure bundling.

2. MODEL

For simplicity, we consider two products with three firms. Firms are denoted by upper case letters products by numbers. Firm A is supposed to produce both product 1 and product 2. It is the only multi-product firm. We further assume that firm B1 produces only product 1 and firm B2 provides only product 2. Firm B1 and firm B2 do not cooperate, thus, they can simply set the stand-alone prices at $P_{B1}$ and $P_{B2}$, respectively. The multi-product firm A can set prices in both markets competing against stand-alone firms B1 and B2. In addition, firm A might offer a bundle. We assume away fixed or variable cost for the production. Therefore, cost reduction effect of bundling is out of analysis.

At the beginning, the multi-product firm A chooses how to compete. The first option is to offer stand-alone prices without offering any discount for the bundle purchase, i.e., ‘linear pricing’ or ‘component pricing.’ In product 1 market, firm A and firm B1 compete in prices while firm A and firm B2 do in product 2 market. The second alternative is to offer only the bundled package without component sales, i.e., ‘pure bundling.’ The multi-product firm supplies the bundle only, which consists of product 1 and product 2. Consumers are not able to buy one from firm A and the other from a
stand-alone firm i.e., ‘mix-and-match’. Unbundling the package for resale is prohibited. The last alternative is to offer a bundle with discount as well as the components with stand-alone prices, i.e., ‘mixed bundling’. After the multi-product firm A decides the mode of pricing strategy, all firms compete in prices simultaneously. The sole objective of the firms is to maximize their own profits.

We assume infinitely many consumers. A consumer buys at most one unit of each product. The products are indivisible. Product usage ensures gross utility \( v_1 \) and \( v_2 \) buying from product 1 and product 2 respectively. Subjective valuations of \( (v_1, v_2) \) across individuals are assumed constant throughout this paper. In addition, we assume that \( (v_1, v_2) \) is necessarily high enough that every single customer ends up with buying both products.\(^1\)

Given the prices, some people buy all the products from firm A, some other people buy from the stand-alone firms B1 and B2, or someone else might ‘mix-and-match’ the products from firm A and a stand-alone firm. What determines the consumer’s purchase here is modeled by ‘product differentiation,’ that is, a consumer has preference for a particular brand, A or B’s. For product \( i \in \{1,2\} \), the consumer indexed by \( x_i \in [0,1] \) dislikes the multi-product firm A’s product \( i \) by \( t_i x_i \) and B1’s product by \( t_i (1-x_i) \) where “transportation cost” \( t_i \) stands for the intensity of “lock-in” or “royalty” to the choice. As \( t_i \) increases, it becomes hard for the customer at \( x_i \) to change the supplier so that the brand preference intensifies and the markets become monopolized locally. Consumer’s preference for the firms is well described by pair \( (x_1, x_2) \in X = [0,1]^2 \). Like Armstrong and Vickers (2008), consumer heterogeneity stems not from the product

\(^1\) Heterogeneous valuation is a typical assumption in monopoly bundling literature. The assumption is hard to accommodate in competitive bundling. In place of valuation heterogeneity, consumers are differentiated in brand preference, following Hotelling (1929).
valuation, \((v_1, v_2)\), but from brand tastes, \((x_1, x_2)\).²

This paper focuses on whether the multi-product firm needs to utilize mixed bundling strategy while it competes with the stand-alone firms. Compared to the stand-alone firms, the multi-product firm is presumed to be advantageous regarding product compatibility. Enhanced product compatibility by the same firm improves customer satisfaction, which means those products are complementary in utility space. As illustrated in Gandal, Markovich, and Riordan (2005), Microsoft Word and Microsoft Excel are good examples. Both programs use the same functional commands in common, and a document prepared by MS-Word is easily transformed to MS-Excel format, and vice versa. Consumers using both products are better off by more than twice since the enhanced inter-product compatibility makes the Office Suite super-additively attractive bundle package. This product complementarity is defined in such a way that the bundle purchase from a multi-product firm adds non-negative utility \(\alpha\) on the gross utility \((v_1 + v_2)\).³ Note that the purchase from single-product firms does not give this super-additive gain.

A \((x_1, x_2)\)-type consumer’s utility is summarized as follows:

\[
\begin{align*}
    u(x_1, x_2) &= \\
    &= \begin{dcases}
         v_1 + v_2 + \alpha - \{t_1 x_1 + t_2 x_2\} & \text{transfer to } A \quad [AA] \\
         v_1 + v_2 - \{t_1 x_1 + t_2 (1 - x_2)\} & \text{transfer to } A \text{ and } B2 \quad [AB] \\
         v_1 + v_2 - \{t_1 (1 - x_1) + t_2 x_2\} & \text{transfer to } B1 \text{ and } A \quad [BA] \\
         v_1 + v_2 - \{t_1 (1 - x_1) + t_2 (1 - x_2)\} & \text{transfer to } B1 \text{ and } B2 \quad [BB]
    \end{dcases}
\]

For the analytical simplicity, the paper makes additional assumptions: The support of consumer type is defined by \(X = [0,1]^2\), the unit rectangle, and \((x_1, x_2)\)’s are uniformly distributed over \(X\), i.e., \((x_1, x_2) \sim \text{Unif}([0,1]^2)\). Secondly, the linear “transportation cost” is fixed at \(t_1 = t_2 = 1\), that is, the intensity of “royalty” to a brand is the same across all the products and it is normalized to 1. Setting the cost at 1 does not seem restrictive since the assumption means that the utility \(v_i\)’s are divided by \(t\) as standardization. Rather, restrictive one is that the “transportation costs” are symmetric between two products, \(t_1 = t_2\). Imposing by the symmetry between two products, we are mainly interested in the symmetric pricing strategy of the stand-alone firms. Lastly, we will

² Chamberlin (1962, pp. 67-69) noticed that the competition with differentiated products is hardly distinguished from the monopolistic behavior. In contemporary modeling terms, the consumer demand of the monopolist is designed by heterogeneous values, \((v_1, v_2)\), however, for competition with differentiated products, brand preferences, \((x_1, x_2)\), are heterogeneous while \((v_1, v_2)\) is held constant. From a particular firm’s point of view, heterogeneous \((x_1, x_2)\) is seemingly equivalent to heterogeneous \((v_1, v_2)\).

³ I will use the terms, ‘complementarity’ and ‘compatibility’ interchangeably unless it causes ambiguity.
confine the equilibrium to the case in which all buyers end up buying both products. This paper does not study the cases in which some buyers do not buy product 1, or product 2, or both.

3. ASYMMETRIC COMPETITION

Throughout this section, single-product firms $B_1$ and $B_2$ charge prices $P_{B_1}$ and $P_{B_2}$, respectively. Single-product firms are independent each other. Provided that a pricing method is chosen by the multi-product firm $A$, all firms set prices simultaneously. Especially in the case of mixed bundling, firm $A$ charges a bundle discount, $\delta$, as well as the component prices $P_{A_1}$ and $P_{A_2}$.

3.1. Linear Component Pricing

We consider the firms’ problem at time $t = 1$. Given component prices $(P_{A_1}, P_{A_2}, P_{B_1}, P_{B_2})$ by the multi-product and stand-alone firms, $(x_1, x_2)$-type consumer’s utility function is defined by

$$u(x_1, x_2) = \begin{cases} 
  v + a - \{x_1 + x_2\} - \{P_{A_1} + P_{A_2}\} & [AA] \\
  v - \{x_1 + (1-x_2)\} - \{P_{A_1} + P_{B_2}\} & [AB] \\
  v - \{(1-x_1) + x_2\} - \{P_{B_1} + P_{A_2}\} & [BA] \\
  v - \{(1-x_1) + (1-x_2)\} - \{P_{B_1} + P_{B_2}\} & [BB] 
\end{cases}$$

where $v = v_1 + v_2$. Figure 1 portrays the types of demands at given prices.

The firms’ profits are

$$\pi_{A} = P_{A_1}(AB + AA) + P_{A_2}(BA + AA),$$

$$\pi_{B_1} = P_{B_1}(BA + BB),$$

$$\pi_{B_2} = P_{B_2}(AB + BB).$$

Suppose that the complementarity is strictly positive, and $P_{A_1} = P_{A_2} = P_{B_1} = P_{B_2} = 1$ tentatively. By raising price $P_{A_1}$ by $\epsilon$, the multi-product firm loses the customers on the margin but gains from the infra-marginal customers. Due to this change, some $A_1$ users switch to $B_1$ ($k_1$ and $k_3$ in Figure 1). Moreover, some customers who would buy
both product 1 and product 2 from the same multi-product firm would switch to $B_1$ and $B_2$ ($k_3$ in Figure 1). Increase in $P_{A1}$ marginally decreases the demand for $A_2$, and the marginal loss is

$$
\varepsilon \left( \frac{dAB}{dP_{A1}} + \frac{dAA}{dP_{A1}} \right) = \varepsilon \left[ \frac{1 - \alpha}{4} + \frac{1 + \alpha}{4} \right] = \varepsilon \frac{1}{2}.
$$

Figure 1. Demands under Linear Component Pricing

Due to the increase in $P_{A1}$, the multi-product firm earns more profits from the infra-marginal customers (AB and AA in Figure 1) by

$$
\varepsilon[AB + AA] = \varepsilon \left[ \frac{2 + 4\alpha + \alpha^2}{8} + \frac{1 - \alpha}{4} \right] = \varepsilon \left[ \frac{1}{2} + \frac{\alpha}{4} + \frac{\alpha^2}{8} \right].
$$

Since the marginal loss is less than the infra-marginal gain, the multi-product firm has an incentive to raise the component price as long as the product complementarity is positive. Formal analysis of the equilibrium is as follows.

Solving the first-order conditions of the firms, we derive the equilibrium price,
\[ P_A^L = P_{A1}^L = P_{A2}^L = \frac{12 + 6\alpha + a^2}{2(6 + a)}, \]  
\[ P_B^L = P_{B1}^L = P_{B2}^L = \frac{12 + 6\alpha + a^2}{(2 + a)(6 + a)}, \]  

and the corresponding profits in equilibrium are determined such as:

\[ \pi_A^L = \frac{(12 + a(6 + a))^2}{2(2 + a)(6 + a)^2}, \]  
\[ \pi_B^L = \pi_{B1}^L = \pi_{B2}^L = \frac{(-12 + a(2 + a))^2}{8(6 + a)^2}, \]

with product complementarity \( a \geq 0 \).

Remarkably, the prices and profits without product complementarity, (i.e., \( a = 0 \)) are:

\[ P_A^L = P_{B1}^L = 1, \  \pi_A^L = 1, \text{ and } \pi_B^L = \frac{1}{2}. \]

where the multi-product firm and the stand-alone firms equally divide markets as shown in Figure 2.

At this symmetric equilibrium, no firm has any incentive to deviate from the symmetric pricing. Similarly to the previous argument, if firm \( A \) decreases \( P_{A1} \) infinitesimally, the incremental change would attract more marginal consumers who would buy product 1 from firm \( A \). However, the change does not attract anyone who buys product 2 from firm \( B2 \). In other words, there is no externality of the price of product 1 on the demand for product 2. Therefore, the multi-product firm cannot take advantage of price coordination of \( P_{A1} \) and \( P_{A2} \), compared to the stand-alone firms.

As the complementarity between two products increases, however, the multi-product firm’s profit rises while the stand-alone single-product firms’ profit declines. It is because more consumers would like to buy both products from firm \( A \) where all the prices are the same. Taking the multi-product firm’s merits into account, all firms compete aggressively, and the prices decline. Meanwhile the multi-product firm charges the prices higher than the stand-alone firms. Since the consumers prefer the enhanced compatibility, the multi-product firm acts less aggressively. Figures 3 and 4 depict the equilibrium prices and profits.
Figure 2. Demands under Linear Pricing when $\alpha = 0$

Figure 3. Prices under Linear Pricing Competition
Proposition 1. Under a component pricing, the multi-product firm has no advantage over the stand-alone single-product firms without merit of product complementarity. Enhanced product complementarity benefits the multi-product firm, but it makes single-product competitors worse off.

Proof. See the appendix.

3.2. Pure Bundling

In this subsection, we consider the case in which the multi-product firm offers the bundle only. No customer is allowed to unbundle the package for resale. The consumers are supposed to choose the firm $A$’s bundle, or they should assemble the products buying separately from the stand-alone firms. Given firm $A$’s bundle price and stand-alone prices, $(P_A, P_{B1}, P_{B2})$, $(x_1, x_2)$-type consumer’s utility function is defined as follows:

\[
u(x_1, x_2) = \begin{cases} 
 v + a - \{x_1 + x_2\} - P_A & [AA] \\
 v - \{(1-x_1) + (1-x_2)\} - \{P_{B1} + P_{B2}\} & [BB]
\end{cases}
\]

where \(v = v_1 + v_2\). Notably, consumers cannot ‘mix-and-match,’ that is, there are no demands for \([AB]\) or \([BA]\) in Figure 1. Figure 5 depicts the types of demands given these
prices. The firms’ profits are
\[\pi_A = P_A \times AA,\]
\[\pi_{B1} = P_{B1} \times BB,\]
\[\pi_{B2} = P_{B2} \times BB,\]
where
\[AA = 1 - \frac{1}{8} (2 - (P_{B1} + P_{B2} - P_A + \alpha))^2,\]

\[\text{Figure 5.} \text{ Demands under Pure Bundling}\]

\[\text{B1 or B2}\]

\[\text{B1 or B2}\]

\[\text{B1 or B2}\]

One might suspect the case in which the demand for bundle from ‘B1+B2’ is greater than the demand for A. It is trivial to prove that this case cannot survive in equilibrium. The demand for [AA] is strictly greater than [BB].
\[ BB = \frac{1}{8} (2 - (P_{b1} + P_{b2} - P_A + \alpha))^2. \]

Suppose that the multi-product firm increases bundle price \( P_A \) by infinitesimal amount \( \varepsilon > 0 \). Due to this price increase, firm \( A \) would lose the customers on the margin (\( k_2 \) in Figure 5), but it makes more profits from the incumbent customers (\( AA \) in Figure 5). The profit loss by the marginal customers is

\[ \frac{\varepsilon}{4} (2 - (P_{b1} + P_{b2} - P_A + \alpha)), \]

while the profit gain from the infra-marginal customers is

\[ \varepsilon \left[ 1 - \frac{1}{8} (2 - (P_{b1} + P_{b2} - P_A + \alpha))^2 \right]. \]

For simplicity, let \( P_A = P_{b1} + P_{b2} \) and \( \alpha = 0 \). Then, the marginal loss is \( \frac{\varepsilon}{2} \) and the infra-marginal gain is \( \frac{3\varepsilon}{4} \). Thus, we conclude that the multi-product firm is incentivized to increase the bundle price greater than the sum of the standalone prices.

In addition, we can immediately infer that firm \( A \) is willing to increase the bundle price as the complementarity enhances. Putting \( \hat{P}_A = P_A - \alpha \), we can transform \( P_A \) into \( \hat{P}_A \) in the marginal loss and the infra-marginal gain equations. Therefore, Given optimal \( \hat{P}_A \), the greater \( \alpha \) is, the larger \( P_A \) is. One can consider the single-product firms’ problem in a similar way.

From the profit maximization problems, we derive the first-order conditions to obtain the symmetric equilibrium of \( P_{b1} \) and \( P_{b2} \):

\[ P_{A}^{PB} = \frac{1}{5} \left( -6 + 3\alpha + 2\sqrt{44 + \alpha(\alpha - 4)} \right), \] (5)

\[ P_{B}^{PB} = P_{b1}^{PB} = P_{b2}^{PB} = \frac{1}{10} \left( 2 - \alpha + \sqrt{44 + \alpha(\alpha - 4)} \right), \] (6)

and the profits in equilibrium are
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\[ \pi_{A}^{PB} = \frac{1}{500} \left( -6 + 3\alpha + 2\sqrt{44 + \alpha(\alpha - 4)} \right) \left( 76 - 2\sqrt{44 + \alpha(\alpha - 4)} + \alpha(4 - \alpha + \sqrt{44 + \alpha(\alpha - 4)}) \right). \quad (7) \]

\[ \pi_{B}^{PB} = \frac{(2 - \alpha + \sqrt{44 + \alpha(\alpha - 4)})^2}{2000}. \quad (8) \]

**Figure 6.** Prices under Pure Bundling Competition

**Figure 7.** Profits under Pure Bundling Competition
The algebraic solutions are displayed as follows. Figure 6 visualizes the pure bundling equilibrium prices, Equations (5) and (6). Figure 7 draws equilibrium profits, Equations (7) and (8).

Note that how the equilibrium plays out with no product complementarity, that is, the multi-product firm has no technological advantage over the single-product firms in regard to consumer preference. When two products are independent, i.e., $\alpha = 0$, Figure 6 shows that multi-product firm $A$ charges the bundle lower than the aggregate prices of the stand-alone single prices, $P_{B1}^{PB} + P_{B2}^{PB}$.

The multi-product firm’s aggressive pricing results from its internalization of “double marginalization.”\(^5\) Suppose that the multi-product firm cuts down the bundle price infinitesimally. By lowering bundle price $P_A$, firm $A$ attracts some customers buying from the stand-alone firms. The converted new customers start to buy both products from firm $A$.

Now, for comparison, suppose that stand-alone firm $B1$ drops its own price $P_{B1}$ infinitesimally. This decrease in $P_{B1}$ would attract some customers to buy product 1. Because of the change in $P_{B1}$, the customer has no choice but to buy product 2 from firm $B2$. This is the external spillover effect of $P_{B1}$ on firm $B2$’s sale ($k_2$ in Figure 5). Likewise, firm $B2$ does not reflect on how its own price $P_{B2}$ affects firm $B1$’s sale ($k_2$ in Figure 5). As long as the single-product firms do not bargain on the positive externality, the stand-alone firms underestimate the marginal effective benefit of their own prices. In this sense, the two standalone firms have the problem of “double marginalization.” The multi-product firm does not have this type of problem since it dispenses only bundled packages. Owing to pure bundling strategy, the multi-product firm has advantage over the stand-alone firms with no product complementarity.

Equations (5) and (6) show the contrasting price movements between the multi-product firm and the single-product firms. As the product complementarity becomes large, the multi-product firm raises the bundle price while the single-product firms lower their prices. The enhanced complementarity favors the multi-product firm over the stand-alone firms. On the one hand, knowing that more customers want to buy from the multi-product firm, firm $A$ is likely to rack up the bundle price. On the other hand, anticipating the multi-product firm’s pricing strategy, the stand-alone firms would put the prices down. These countervailing forces boil down to the equilibrium prices which make the multi-product firm benefit and the stand-alone firms lose from the enhanced

\(^5\) The name of “double marginalization” originates from the vertical integration by the successive monopolists. If the upstream and the downstream firms charge their prices independently, then the final goods price is higher than that of the vertically integrated monopolist. This phenomenon is generalized where two monopolists independently set prices for complementary goods. Refer to Spengler (1950).
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complementarity. Nalebuff (2000) studies a monopolistic pure bundling with potential entry and he arrives at the similar results.

**Proposition 2.** With a pure bundling, the multi-product firm earns more than the aggregate of the single-product firms in total. As the product complementarity gets fortified, the multi-product firm gets better off while the single-product firms become worse off.

Proof. See the appendix.

### 3.3. Mixed Bundling

Mixed bundling is the combination of component pricing and pure bundling. On top of component prices, \( (P_A^1, P_A^2) \), multi-product firm \( A \) has one more choice variable: A bundle discount \( \delta \). As before, single-product firms ask their own prices, \( P_B^1 \) and \( P_B^2 \), non-cooperatively.

Given prices \( (P_A^1, P_A^2, \delta; P_B^1, P_B^2) \), \((x_1, x_2)\)-type consumer’s utility is defined by the following:

\[
\begin{align*}
    u(x_1, x_2) = & \begin{cases} 
        v + a - (x_1 + x_2) - (P_A^1 + P_A^2 - \delta) & [AA] \\
        v - (x_1 + (1 - x_2)) - (P_A^1 + P_B^2) & [AB] \\
        v - ((1 - x_1) + x_2) - (P_B^1 + P_A^2) & [BA] \\
        v - ((1 - x_1) + (1 - x_2)) - (P_B^1 + P_B^2) & [BB] 
    \end{cases}
\end{align*}
\]

Consumers are classified into four different types according to which component they buy from which seller. Figure 8 illustrates the partition of consumers given the prices.

The profits of the firms are defined by

\[
\begin{align*}
    \pi_A &= P_A^1(AB + AA) + P_A^2(BA + AA) - \delta AA, \\
    \pi_B^1 &= P_B^1(BA + BB), \\
    \pi_B^2 &= P_B^2(AB + BB),
\end{align*}
\]
given bundle discount as well as component prices.
In Section 3.1 and 3.2, we contemplate prices to infinitesimally deviate from the first-order conditions. Here we can apply the same logic. For the case of the multi-product firm, we have three equations that balances the marginal loss and the infra-marginal gain with $P_{A1}$, $P_{A2}$, and discount $\delta$. From the standalone firms, we derive two first-order conditions that equates the marginal loss and the infra-marginal gain with $P_{B1}$ and $P_{B2}$, respectively. Compared to the linear component pricing in Section 3.1, the new introduction of bundle discount $\delta$ complicates the solution that we needs to depend heavily on the numerical analysis. Imposing symmetry such as $P_{A1} = P_{A2}$ and $P_{B1} = P_{B2}$, we collapse all the first-order conditions into one polynomial equations higher than fourth degree, hence no algebraic solution in general by Abel’s impossibility theorem. Instead we numerically solve for equilibrium prices and profits. Detailed steps for the numerical analysis can be verified in the appendix.

Confining to non-negative real-valued solutions, we have three mixed bundling equilibria within a limited range of product complementarity. One is a linear component pricing that we have examined in Section 3.1, in which bundle discount $\delta$ is zero for any complementarity $\alpha \geq 0$. It is a trivial mixed bundling. The other two mixed bundling equilibria are non-trivial. Naming equilibriums (1) and (2), we interpret the patterns with numerical solutions.

Figures 9 and 10 show the equilibrium price strategies and profits under equilibrium

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6 Refer to Fraleigh (1999) for a rigorous proof.
(1) while Figures 11 and 12 indicate equilibrium (2).

**Figure 9.** Prices under Mixed Bundling Equilibrium (1)

**Figure 10.** Profits under Mixed Bundling Equilibrium (1)
As shown in Figure 9, equilibrium (1) appears eccentric in the sense of prices. Where the products are independent, i.e., no product complementarity, the multi-product firm offers a bundle discount greater than its stand-alone prices. Since the bundle is cheaper than a single product, no customer buys only one product from firm $A$. Therefore customers buy both products either from the multi-product firm or from the single-product firms.\(^7\) In terms of the profits depicted by Figure 10, equilibrium (1) is hardly acceptable since the multi-product firm cannot perform worse than the single-product firms in terms of profit. As the complementarity improves, the multi-product firm benefits gradually while the single-product firm gets a loss. Notably, however, the aggregate profit of the stand-alone firms exceeds the multi-product firm’s profit for any value of complementarity.

With regard to the prices, equilibrium (2) seems more natural than equilibrium (1). As shown in figure 11, stand-alone prices of the multi-product firm are higher than the bundle discount. Figure 12, yet, provides such a paradox as Figure 10: The multi-product firm earns less than the stand-alone firms. Even more, the multi-product firm loses the profit as the complementarity becomes strengthened. Details for numerical computations will be found in the appendix.

\(^7\) Someone might be concerned about the case in which a consumer buys the bundle from firm $A$ to get a single product he really wants. For example, the consumer buys product 1 from firm $B1$ and the bundle from firm $A$. He does buy the bundle to single out product 2 in fact. Therefore we need no “free disposal” assumption in this paper. If you are interested in the disposal-proof bundle offer, equilibrium (1) will be confined to where $\delta < \phi^{MB}_A$. 

![Figure 11. Prices under Mixed Bundling Equilibrium (2)](image-url)
Proposition 3. There exist two non-trivial mixed bundling equilibria within the limited value of complementarity. In both equilibria, the single-product firms make more profit aggregately than the multi-product firm does.

3.4. Comparison

Consider the multi-product firm’s optimal choice of competition mode at the beginning. Firm $A$ can choose either ‘linear component pricing,’ ‘pure bundling,’ or ‘mixed bundling’ depending on the expected profit that follows. Figure 13 compares all the profits that the multi-product firm expects.

As noted in proposition 3, we have multiple mixed bundling equilibria that are somewhat strange. Figure 13 shows that non-trivial mixed bundling equilibria are dominated by component pricing or pure bundling in terms of profits. Therefore, those mixed bundling equilibria will not be chosen by multi-product firm at the beginning since it is strictly dominated by other pricing strategies. Now the plausible alternatives for the multi-product firm are either ‘linear component pricing’ or ‘pure bundling.’ The former is preferred where the product-complementarity is relatively low. More precisely, the multi-product firm prefers the linear component pricing to the pure bundling if the complementarity is approximately less than equal to 0.3050 in this simple model. Otherwise, pure bundle offer is preferred.
Figure 13. Comparison of Profits under Different Mode of Competition: Multi-product Firm A

Figure 14. Comparison of Profits under Different Mode of Competition: Single-product Firm B
**Proposition 4.** Mixed bundling strategy is strictly dominated by either component pricing or pure bundling. The multi-product firm prefers pure bundling to component pricing where the product complementarity is greater than 0.3050. Otherwise, linear component pricing will be chosen.

The corresponding single-product firm’s profit is depicted in Figure 14. Single-product firm’s profit deteriorates as firm A’s product-complementarity gets improved. At a threshold value, \( a = 0.3050 \), the profit drops sharply as the competition mode switches from ‘linear pricing’ to ‘pure bundling.’

4. CONCLUDING REMARKS

Adams and Yellen (1976) argue that “whenever the exclusion requirement\(^8\) is violated in a pure bundling equilibrium, mixed bundling is necessarily preferred to pure bundling.” Lewbel (1985) confirms the results even taking into account product complementarity. Dominance of the mixed bundling is intuitive in monopoly bundling literature. When it pertains to the competitive bundling, however, we are not assured that the mixed bundling dominates the pure bundling or the component pricing.

Armstrong and Vickers (2009) verify that a two-part tariff type of bundle discount is a viable equilibrium strategy even in symmetric competition between two multi-product firms. This paper departs from Armstrong and Vickers (2009) by assuming the multi-product firm to compete with single-product firms in each product market. PC office software market is the prominent example empirically studied by Gandal, Markovich, and Riordan (2005), which stipulates that word processors and spreadsheets are complementary products and the mixed bundling coincides with the complementarity.

In contrast to the previous literature, this paper concludes that the product complementarity does not lead to the use of mixed bundling. A simple duopoly model of asymmetric competition eliminates the necessity of mixed bundling. As proposition 4 states, mixed bundling does not benefit the multi-product firm no matter how strongly the products are compatible.

There remain open questions for further research. I assume the cases in which every customer ends up with buying all products. Yet, if prices are high enough, someone will buy a single product and some others will not buy any product at all. A sophisticated analysis is expected to account for the consumer’s participation. Another interesting line of research would be the study on the stand-alone firms’ cooperation to offer a bundle. They might have incentive for merger and acquisition, or simply form a business coalition while drawing a contract. Lastly, we need to do social welfare analysis for designing an efficient regulation.

\(^8\) No individual consumes a good if the cost of that good exceeds its cost in fact consumes that good.
APPENDIX

Proof of Proposition 1.

The firms’ profits are

$$\pi_A = P_{A1}(AB + AA) + P_{A2}(BA + AA),$$  \hspace{1cm} (A1)

$$\pi_{B1} = P_{B1}(BA + BB),$$  \hspace{1cm} (A2)

$$\pi_{B2} = P_{B2}(AB + BB),$$  \hspace{1cm} (A3)

where

$$AA = \frac{1}{8}(2 + 4a + a^2 + 2P_{B1} + 2aP_{B1} - 2P_{A2}(1 + a + P_{B1}) + 2P_{B2}(1 + a + P_{B1})) \hspace{1cm} (A4)$$

$$AB = \frac{1}{4}((-1 + P_{A1} - P_{B1})(-1 + a - P_{A2} + P_{B2})), \hspace{1cm} (A5)$$

$$BA = \frac{1}{4}((-1 + P_{A2} - P_{B2})(-1 + a - P_{A1} + P_{B1})), \hspace{1cm} (A6)$$

$$BB = \frac{1}{8}(2 - a^2 - 2P_{A2}(-1 + P_{B1}) - 2P_{B1} + 2P_{A1}(1 + P_{A2} - P_{B2}) + 2P_{B2}(-1 + P_{B1})). \hspace{1cm} (A7)$$

as referred to in figure 1. Then the first-order conditions of profit maximization are

$$(FOC : P_{A1}) : \frac{1}{8}(4 + a(2 + a) - 8P_{A1} - 4aP_{A2} + 4P_{B1} + 2aP_{B2}) = 0, \hspace{1cm} (A8)$$

$$(FOC : P_{A2}) : \frac{1}{8}(4 + a(2 + a) - 8P_{A2} - 4aP_{A1} + 4P_{B2} + 2aP_{B1}) = 0, \hspace{1cm} (A9)$$

$$(FOC : P_{B1}) : \frac{1}{8}(4 - a(2 + a) + 4P_{A1} + 2aP_{A2} - 8P_{B1} - 2aP_{B2}) = 0, \hspace{1cm} (A10)$$
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\[(FOC: P_{b2}^L) : \frac{1}{8} (4 - \alpha(2 + \alpha) + 4P_{A2} + 2\alpha P_{A1} - 8P_{B2} - 2\alpha P_{B1}) = 0 . \]  \hfill (A11)

So the equilibrium prices are determined by

\[P_A^L = P_{A1}^L = P_{A2}^L = \frac{12 + 6\alpha + \alpha^2}{2(6 + \alpha)}, \]  \hfill (A12)

\[P_B^L = P_{B1}^L = P_{B2}^L = \frac{12 + 6\alpha + \alpha^2}{(2 + \alpha)(6 + \alpha)}, \]  \hfill (A13)

and the profits in equilibrium are pinned down

\[\pi_A^L = \frac{(12 + \alpha(6 + \alpha))^2}{2(2 + \alpha)(6 + \alpha)^2}, \]  \hfill (A14)

\[\pi_B^L = \pi_{B1}^L = \pi_{B2}^L = \frac{(-12 + \alpha(2 + \alpha))^2}{8(6 + \alpha)^2}, \]  \hfill (A15)

at a certain product complementarity \( \alpha \geq 0 \).

Besides the first-order conditions, we can immediately verify that the second-order sufficient conditions are satisfied.

**Proof of Proposition 2.**

The firms’ profits are

\[\pi_A = P_A \times AA, \]  \hfill (A16)

\[\pi_B1 = P_{B1} \times BB, \]  \hfill (A17)

\[\pi_B2 = P_{B2} \times BB, \]  \hfill (A18)

where

\[AA = 1 - \frac{1}{8} (-2 - P_A + P_{B1} + P_{B2} + \alpha)^2, \]  \hfill (A19)
\[ BB = \frac{1}{8}(-2 - P_A + P_{b1} + P_{b2} + \alpha)^2, \quad (A20) \]

as referred to in figure 5. Then the first-order conditions of profit maximization are

\[
(FOC : P_A): \quad \frac{1}{8}(-3P_A^2 + P_A(-2 + P_{b1} + P_{b2} + \alpha) - (P_{b1} + P_{b2} + \alpha)^2 \\
+ 4(1 + P_{b1} + P_{b2} + \alpha)) = 0, \quad (A21)
\]

\[
(FOC : P_{b1}): \quad \frac{1}{8}(2 + P_A - 3P_{b1} - P_{b2} - \alpha)(2 + P_A - P_{b1} - P_{b2} - \alpha) = 0, \quad (A22)
\]

\[
(FOC : P_{b2}): \quad \frac{1}{8}(2 + P_A - 3P_{b2} - P_{b1} - \alpha)(2 + P_A - P_{b1} - P_{b2} - \alpha) = 0. \quad (A23)
\]

So the equilibrium prices are determined by

\[
P_{A}^{PB} = \frac{1}{5}(-6 + 3\alpha + 2\sqrt{44 + \alpha(\alpha - 4)}), \quad (A24)
\]

\[
P_{b1}^{PB} = P_{b2}^{PB} = \frac{1}{10}(2 - \alpha + \sqrt{44 + \alpha(\alpha - 4)}), \quad (A25)
\]

and the profits in equilibrium are

\[
\pi_{A}^{PB} = \frac{1}{5000}\left(-6 + 3\alpha + 2\sqrt{44 + \alpha(\alpha - 4)}\right) \\
\left(76 - 2\sqrt{44 + \alpha(\alpha - 4)} + \alpha(4 - \alpha + \sqrt{44 + \alpha(\alpha - 4)})\right), \quad (A26)
\]

\[
\pi_{b1}^{PB} = \pi_{b2}^{PB} = \frac{(2 - \alpha + \sqrt{44 + \alpha(\alpha - 4)})^2}{2000}, \quad (A27)
\]

at a certain product complementarity \(\alpha \geq 0\).

At the equilibrium prices, we can easily verify that the second-order condition is satisfied.

**Numerical Steps for Proposition 3.**

The profits of the firms are defined by
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\[ \pi_A = P_{A1}(AB + AA) + P_{A2}(BA + AA) - \delta AA, \]  
(A28)

\[ \pi_{B1} = P_{B1}(BA + BB), \]  
(A29)

\[ \pi_{B2} = P_{B2}(AB + BB), \]  
(A30)

where

\[ AA = -\frac{1}{8}(\alpha + \delta)^2 + \frac{1}{4}(1 - P_{A1} + P_{B1} + \alpha + \delta)(1 - P_{A2} + P_{B2} + \alpha + \delta), \]  
(A31)

\[ AB = \frac{1}{4}(1 - P_{A1} + P_{B1})(1 + P_{A2} - P_{B2} - \alpha - \delta), \]  
(A32)

\[ BA = \frac{1}{4}(1 - P_{A2} + P_{B2})(1 + P_{A1} - P_{B1} - \alpha - \delta), \]  
(A33)

\[ BB = \frac{1}{8}(2 - 2P_{A2}(-1 + P_{B1}) + 2P_{B1}(1 + P_{A2} - P_{B2}) \]
\[ + 2P_{B1}(-1 + P_{B2}) - 2P_{B2} - (\alpha + \delta)^2). \]  
(A34)

The first-order conditions of profit maximization are

\[ (FOC: P_{A1}): \frac{1}{8}(4 - 8P_{A1} + 4P_{B1} + \alpha^2 + \delta(4 - 6P_{A2} + 4P_{B2} + 3\delta) \]
\[ + \alpha(2 - 4P_{A2} + 2P_{B2} + 4\delta)) = 0, \]  
(A35)

\[ (FOC: P_{A2}): \frac{1}{8}(4 - 8P_{A2} + 4P_{B2} + \alpha^2 + \delta(4 - 6P_{A1} + 4P_{B1} + 3\delta) \]
\[ + \alpha(2 - 4P_{A1} + 2P_{B1} + 4\delta)) = 0, \]  
(A36)

\[ (FOC: \delta): \frac{1}{8}(-2 - 2P_{B1} - 2P_{B2} - 2P_{B1}P_{B2} - 4\alpha - 2P_{B1}(\alpha - 2P_{B2}a - \alpha^2 \]
\[ - 4(2 + P_{B1} + P_{B2} + \alpha)\delta - 3\delta^2 + P_{A2}(4(1 + P_{B1} + \alpha + 6\delta) \]
\[ + P_{A1}(-6P_{A2} + 4(1 + P_{B2} + \alpha) + 6\delta) = 0, \]  
(A37)
(FOC: \(P_{b1}\)) \[ \frac{1}{8} (4 + 4P_{d1} - 8P_{b1} - \alpha(2 - 2P_{d2} + 2P_{b2} + \alpha) - 2\delta 
\] 
\[ - 2(-P_{d2} + P_{b2} + \alpha)\delta - \delta^2 = 0, \] \hspace{1cm} (A38)

(FOC: \(P_{b2}\)) \[ \frac{1}{8} (4 + 4P_{d2} - 8P_{b2} - \alpha(2 - 2P_{d1} + 2P_{b1} + \alpha) - 2\delta 
\] 
\[ - 2(-P_{d1} + P_{b1} + \alpha)\delta - \delta^2 = 0. \] \hspace{1cm} (A39)

All of the first-order conditions other than (FOC: \(\delta\)) are decreasing in prices. However, the first-order condition (FOC: \(\delta\)) is a quadratic function of \(\delta\). Thus we need to verify whether the interior solutions satisfy the second-order condition. Besides the interior solution, we need to investigate the corner solution when \(\delta = 0\). Comparing the first-order conditions of linear pricing with those of mixed bundling, the corner solution is equivalent to the linear component pricing equilibrium. Therefore, I will focus on the interior solutions satisfying the first-order conditions associated with four demand types exist.

The derivative of (FOC: \(\delta\)) is

\[ (FOC: \delta) = \frac{1}{4} (3P_{d1} + 3P_{d2} - 2(2 + P_{b1} + P_{b2} + \alpha) - 3\delta) < 0. \] \hspace{1cm} (A40)

Solving first-order conditions except for (FOC: \(\delta\)), we obtain

\[ P_{d1} = P_{d2} = \frac{\delta}{2} + \frac{6(2 + \delta) + \alpha(6 + \alpha + 2\delta)}{12 + \alpha^2 + 2\alpha(4 + \delta) + \delta(10 + \delta)}, \] \hspace{1cm} (A41)

\[ P_{b1} = P_{b2} = -\frac{\delta}{2} + \frac{2(2 + \delta)(3 + \alpha) + (8 + 3\alpha)\delta}{12 + \alpha^2 + 2\alpha(4 + \delta) + \delta(10 + \delta)}. \] \hspace{1cm} (A42)

Plugging Equations (A41) and (A42) into (FOC: \(\delta\)), we can simplify the condition (FOC: \(\delta\)) as

\[ 0 = 576 + 576\alpha - 48\alpha^2 - 48\alpha^3 + 32\alpha^4 + 12\alpha^5 + \alpha^6 - 768\alpha\delta - 192\alpha^2\delta \]
\[ + 256\alpha^3\delta + 92\alpha^4\delta + 8\alpha^5\delta - 912\delta^2 - 336\alpha\delta^2 \]
\[ + 616\alpha^2\delta^2 + 256\alpha^3\delta^2 + 25\alpha^4\delta^2 - 192\delta^3 + 608\alpha\delta^3 \]
\[ + 336\alpha^2\delta^3 + 40\alpha^3\delta^3 + 312\delta^4 + 212\alpha\delta^4 + 35\alpha^4 \]
\[ + 52\delta^5 + 16\alpha\delta^5 + 3\delta^6, \] \hspace{1cm} (A43)
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with respect to $\delta$. Roots of polynomial (A43) will be found numerically.

The solutions are confined to non-negative real valued $\delta$ which supports the existence of the equilibrium. Figures 9 to 12 portray the numerical solutions satisfying the second-order conditions as well as the first-order conditions.

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