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PORTFOLIO SELECTION USING NEW FACTORS BASED ON FIRM CHARACTERISTICS*

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In this paper, we apply a new factor model to portfolio-selection problems and compare its portfolio investment performance with those of other popular portfolio-selection methods. The new factors are formed from a well-characterized subset of the asset universe based on firm characteristics and exhibit better asset-pricing performance than popular extant asset-pricing factors. The performance comparison shows that the new factors exhibit better portfolio investment performance than alternative methods for various test portfolios and various periods.

Keywords: Portfolio Selection, Asset Pricing Models, Mean-Variance Analysis, Sharpe Ratio, Firm Characteristics
JEL Classification: C22; C23; G11; G15

1. INTRODUCTION

Portfolio selection has been a core research area in finance since the invention of the Markowitz (1952) mean-variance framework. Despite significant developments in this field, the performance of existing portfolio-selection methods is not yet satisfactory, mainly due to the multifaceted requirements of actual portfolio-selection problems.

One important problem occurs when many assets are included as investment assets. As more assets are considered for portfolio investment, many more parameters must be estimated. This "curse of dimension" inevitably leads to inaccurate estimations and thus a poor portfolio investment performance.[1] This problem is often addressed using the

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[1] Many other important issues are also associated with actual portfolio-selection problems. For example, it is difficult to perform multiperiod portfolio selection with many assets. The extension of usual preferences to incorporate behavioral anomalies is another potential issue. Furthermore, this approach also has important implications for accounting for random investment horizons, market friction, and background risks. See

factor approach,[2] which assumes that a small number of factors can succinctly describe the dynamics of many asset returns. Thus, the direct estimation of the first and second moments of many expected asset returns is avoided, greatly enhancing the estimation accuracy and, ideally, improving the portfolio investment performance. The success of the factor approach significantly depends upon the quality of the factors used. The most popular factors for portfolio selection have been chosen among the asset-pricing factors that have proven to successfully account for multiple asset returns.[3] This convention is understandable, as we expect quite similar roles for factors in both asset-pricing and portfolio-selection problems.

Recently, Suh, Song, and Lee (2014) proposed a new method to form well-diversified portfolios as asset-pricing factors. These new factors are formed from a well-characterized subset of the asset universe, called basis assets. These basis assets are structured based on firm characteristics found by previous studies to be useful for explaining (co)variation in asset returns. The new factors have several advantages. For example, instead of arbitrarily selecting a small number of firm characteristics among many candidates, the new factors consider all of these firm characteristics. Moreover, the new factors are constructed in such a way as to account for both time-series and cross-sectional variations of asset returns. Because the new factors can flexibly incorporate new firm characteristics, they will better represent the asset universe as more firm characteristics are found in the future. Suh, Song, and Lee (2014) apply the new factors to U.S. equity return data and show empirical results that the new factors exhibit better asset-pricing performance than popular extant asset-pricing factors do.

In this paper, we apply the new factors proposed by Suh, Song, and Lee (2014) to portfolio-selection problems and compare portfolio investment performances with other popular asset-pricing factors and also other portfolio-selection methods. The performance comparison results indicate that the new factors exhibit better portfolio investment performance than alternative methods for various test portfolios and various periods.

The rest of the paper is organized as follows. Section 2 explains the method of

Brandt (2010) for a survey and relevant references.

[2] In addition to the factor approach, other approaches have been proposed. For example, the shrinkage estimation approach is simple to implement but provides significant improvement in many cases (see, e.g., James and Stein, 1961; Jobson et al., 1979; Jobson and Korkie, 1981; Frost and Savarino, 1986; Jorion, 1986; DeMiguel et al., 2013; among others). On the other hand, a decision-theoretic approach explicitly takes into account parameter uncertainty and informative priors. Zellner and Chetty (1965), Klein and Bawa (1976), and Brown (1978) are among the first to apply the decision-theoretic approach to portfolio-selection problems. Black and Litterman (1992) and Pástor (2000) attempt to use economic theories as informative priors. Recently, Brandt et al. (2009) parameterize the portfolio weights as a function of observable variables (see Hjalmarsson and Manchev (2012) for a related analysis).

[3] Another way of finding factors for portfolio selection is to use statistical factor analysis (e.g., Roll and Ross (1980) and Connor and Korajczyk (1986)).

finding new factors. Section 3 discusses the data and the new factors. It also describes the extant asset-pricing models to be compared. The empirical results of the model performance comparison are provided in Section 4. Section 5 concludes.

2. METHODOLOGY

In this section, we explain the method used to form the new factors from firm-characteristics-based portfolios. This method generates two classes of factors: time-series factors and a cross-sectional factor. The time-series factors aim to capture mainly time-series variations of asset returns, while the cross-sectional factor intends to capture cross-sectional variations of asset returns. We then discuss several popular extant asset-pricing factors for comparison purposes. We also discuss the problem of portfolio selection with factors.

2.1. Method to Construct New Factors

For expositional purposes, we briefly explain the method used to form the new factors proposed by Suh, Song, and Lee (2014). The new method attempts to construct factors from a subset called basis assets rather than the asset universe. This restriction greatly facilitates factor formation at the cost of potential information loss. To minimize this cost, it is necessary to form appropriate basis assets. Good basis assets are expected to represent the characteristics of the asset universe well. Details about how to construct the basis assets will be discussed in the next section.

We intend to form factors from linear combinations of the chosen n basis assets. For this purpose, we will employ principal component analysis (PCA). PCA conducts an orthogonal transformation of the data matrix to convert possibly correlated variables into linearly uncorrelated variables called principal components, which can be regarded as factor realizations in a multifactor framework. Moreover, because the principal components are sorted according to their explanatory power, we select the first K principal components as a subset (denoted by f_1) of the new factors. To be concrete, we denote $\mathbf{r}$ as the $T \times n$ excess return data matrix of the basis asset returns with T periods and Ω as the $n \times n$ cross-product of $\mathbf{r}$, i.e., $\Omega = \mathbf{r}'\mathbf{r}/T$. The new factors f_1 will be constructed as weighted averages of n basis assets, where the first K eigenvectors of Ω serve as the weights. These first K principal components basically account for both the time-series and cross-sectional variations of the data matrix by assigning equal weights to the two-dimensional variations. However, because time-series variations are typically much greater than cross-sectional variations in the well-diversified portfolio return data, these factors match the time-series variations better than the cross-sectional variations; thus, we call these factors the time-series (TS) factors.

The TS factors alone might not yield satisfactory results in terms of cross-section performance as measured by, for example, the HJ-distance measure or the cross-sectional regression R^2 measure because the TS factors are constructed to mainly capture the time-series variations.[4] Accounting for cross-sectional variations of asset returns is important for not only asset pricing but also portfolio selection. Successfully accounting for cross-sectional variations may yield accurate estimates of asset correlations and thereby good portfolio investment performance. To improve the ability to capture cross-sectional variations, the new method suggests finding an additional factor in the presence of the K TS factors. We denote by P the total n principal components of $\mathbf{r}$. Because we select the TS factors f_1 as the first K principal components of $\mathbf{r}$, f_1 is the first K columns of P, denoted by p_1. To construct the cross-sectional (CS) factor, we now focus on the cross-sectional pricing error e, which is defined as $e \equiv E[g'y] - 1_n$, where g is a vector of gross returns on n basis assets, y is the stochastic discount factor, and 1_n denotes an n vector of ones. Importantly, we can obtain zero cross-sectional pricing errors for n basis assets by appropriately choosing one additional factor, the single cross-sectional factor. We denote by p_2 the remaining $n-K-1$ principal components by excluding the last principal component because of the inclusion of constant term in the stochastic discount factor y. In the presence of the TS factors $f_1(=p_1)$, the time-series realization of the stochastic discount factor y is specified as

$$\hat{y} = \gamma_0 + p_1\gamma_1 + p_2\gamma_2 \equiv D\gamma, \tag{1}$$

where $D \equiv [1, p_1, p_2]$ and $\gamma = [\gamma_0, \gamma_1', \gamma_2']'$. Now, by simply imposing zero cross-sectional pricing errors on n basis assets, the coefficient γ can be determined; that is, from the zero pricing error condition

$$e \equiv E[g'y] - 1_n = \frac{1}{T}G'D\gamma - 1_n = 0_n, \tag{2}$$

we obtain the estimate of γ as follows:

$$\hat{\gamma} = T[G'D]^{-1}1_n, \tag{3}$$

where 0_n denotes an n vector of zeros and G is the data matrix of gross returns. Our proposed cross-sectional factor is then formed as an average of the principal components

[4] The Hansen-Jagannathan (HJ) distance was proposed by Hansen and Jagannathan (1997) and has been widely used for model diagnosis and selection (e.g., Jagannathan and Wang (1996), Jagannathan, Kubota, and Takehara (1998), Campbell and Cochrane (2000), Lettau and Ludvigson (2001), Hodrick and Zhang (2001), and Chen and Ludvigson (2004)). Another popular goodness-of-fit measure for a model is the cross-sectional R^2, which is analyzed by Kandel and Stambaugh (1995) and Kan, Robotti, and Shanken (2013).

p_2 with weights $\hat{\gamma}_2$ (the estimate of γ_2); that is, the cross-sectional factor is a linear combination of the remaining $n - K - 1$ principal components, specified as follows:

$$f_2 = p_2\hat{\gamma}_2. \quad (4)$$

Notice that we need only one CS factor to make the cross-sectional pricing errors equal zero, whereas we need multiple TS factors.

Some remarks about the new method are in order. First, fundamental factor models (e.g., the Fama-French three-factor model) directly construct factor realizations by choosing some firm characteristics, sorting the cross-section of assets based on the characteristics, and then forming a hedge portfolio that is long in the top quintile and short in the bottom quintile. As more relevant firm characteristics are discovered, more factors should be added; otherwise, some factors should be arbitrarily chosen. Similarly to fundamental factor models, the new method utilizes firm characteristics to construct factors; however, it does not necessarily increase the number of factors, even if more firm characteristics are to be taken into account.

Second, the new method differs from the usual statistical factor models in several ways. Factors under statistical factor models are unobservable and to be extracted from asset returns; thus, it is difficult to link statistical factors with firm characteristics or economic variables. In contrast, the new factors are directly linked with firm characteristics because the basis assets are formed from firm characteristics. In addition, the new method takes into account both TS and CS variations and includes a single CS factor, which also makes the new method different from the usual PCA.

Third, the new method utilizes the PCA to form new factors. Factor analysis is an alternative statistical method for that purpose. We choose the PCA by considering that typical algorithms for factor analysis are not efficient for very large problems and that traditional factor analysis is only appropriate with strong assumptions of cross-sectionally uncorrelated, serially uncorrelated, and serially homoscedastic disturbances.

2.2. Extant Asset-Pricing Factors

For the performance comparison of the new factor model with other extant models, we will consider several popular extant asset-pricing models. Specifically, we select three asset-pricing models, described as follows.

The first model is the CAPM of Sharpe (1964), Lintner (1965), and Black (1972), which specifies the excess return of ith asset at time t as

$$r_{i,t} = \alpha_i + \beta_{MKT,i} r_{MKT,t} + \epsilon_{i,t}^{CAPM}, \quad (5)$$

where $r_{MKT,t}$ denotes the excess return of the market portfolio at time t.

The second model is the three-factor model (FF3) of Fama and French (1993), where the excess return of ith asset at time t is specified as

$$r_{i,t} = \alpha_i + \beta_{MKT,i} r_{MKT,t} + \beta_{SMB,i} r_{SMB,t} + \beta_{HML,i} r_{HML,t} + \epsilon_{i,t}^{FF3}, \tag{6}$$

where $r_{SMB,t}$ denotes the return difference between portfolios of small and large stocks, and $r_{HML,t}$ is the return difference between portfolios of high and low book-to-market ratios.

The third model is the four-factor model (FF4) combining the Fama and French (1993) three-factor model with Carhart's (1997) momentum factor, under which the excess return of ith asset at time t is specified as

$$r_{i,t} = \alpha_i + \beta_{MKT,i} r_{MKT,t} + \beta_{SMB,i} r_{SMB,t} + \beta_{HML,i} r_{HML,t} + \beta_{MOM,i} r_{MOM,t} + \epsilon_{i,t}^{FF4}, \tag{7}$$

where $r_{MOM,t}$ denotes the return difference between portfolios of winner and loser stocks.

2.3. Portfolio Selection with Factors

Suppose we consider an asset universe consisting of N risky assets (which differ from the n basis assets) and a riskless asset. In a mean-variance framework, the optimal portfolio of the risky assets, the "tangency portfolio," is the portfolio with the maximum Sharpe ratio among all portfolios of the N risky assets. If we denote by z_t the vector of excess returns of these N risky assets at time t with mean μ and covariance matrix V and denote by x_N^* the tangency portfolio weight, then a standard result from the mean-variance framework specifies that

$$x_N^* = (1_N V^{-1} \mu)^{-1} V^{-1} \mu. \tag{8}$$

In a linear multifactor model, the risky asset returns are specified as

$$z_t = \alpha + \beta z_{F,t} + \epsilon_t, \tag{9}$$

$$E[\epsilon_t] = 0, \;\; E[\epsilon_t \epsilon_t'] = \Sigma, \tag{10}$$

$$E\left[z_{F,t}\right] = \mu_F, \;\; E\left[z_{F,t} z_{F,t}{}'\right] = \Sigma_F, \tag{11}$$

where $z_{F,t}$ indicates a K vector of factors, α is an N vector, β is an $N \times K$ matrix, Σ is an $N \times N$ matrix, μ_F is a K vector, and Σ_F is a $K \times K$ matrix. Under the above linear multifactor model, the mean and covariance matrix are derived as

$$\mu = \alpha + \beta \mu_F, \tag{12}$$

$$V = \beta\Sigma_F\beta' + \Sigma. \quad (13)$$

By substituting Eqs. (12) and (13) into Eq. (8), we obtain the tangency portfolio weights x_N^* in the linear multifactor model.[5]

3. DATA AND PRICING FACTORS

3.1. Test Portfolios

The out-of-sample performance of portfolio investment will be measured with actual test portfolios. For robustness, data availability, and representativeness, we choose four sets of 26 test portfolios: equal-weighted market portfolio (EW) plus (i) 25 value-weighted portfolios (SIZE×BM; combinations of size quintiles and book-to-market quintiles), (ii) 25 size and short-term-reversal combinations (SIZE×STR), (iii) 25 size and momentum combinations (SIZE×MOM), and (iv) 25 size and long-term-reversal combinations (SIZE×LTR).[6] Portfolio sorts are used extensively throughout finance to establish and test for systematic cross-sectional patterns in expected stock returns related to firm or stock characteristics. While sorts on a single characteristic, such as firm size, are very common, double sorts on multiple characteristics are also widely used and can represent the actual asset universe well.[7]

Table 1 shows summary statistics of the four sets of test portfolios during the sample period from July 1936 to December 2013 for the SIZE×BM and SIZE×STR test portfolio sets, from January 1938 to December 2013 for the SIZE×MOM test portfolio sets, and from January 1950 to December 2013 for the SIZE×LTR test portfolio sets. The mean excess returns are differentiated among portfolios within the same test portfolio set. Panel B shows that the test portfolios exhibit similar levels of average correlations within and between classes, implying that the test portfolio sets are complementary to one another.

[5] MacKinlay and Pástor (2000) derive the tangency portfolio weights under the assumption that factor portfolios are also included as investable assets. In contrast, we do not include factor portfolios as investable assets.

[6] MacKinlay and Pástor (2000) document evidence that the EW passive portfolio performs well compared to other portfolio-selection methods. To improve portfolio performance and take into account this evidence, we include the EW in the test portfolios. All double-sort test portfolio data are available on Kenneth French's web page.

[7] See, for example, Fama and French (1993) for double sorts on size and book-to-market ratio and Rouwenhorst (1998) for size and momentum, among others.

Table 1. Summary Statistics of Test Portfolios and Their Correlations

A. Summary Statistics

Test portfolio	Statistic	Min	Median	Max
SIZE×BM	Mean	0.460	0.860	1.300
	S.D.	4.390	5.810	8.890
	Skewness	-0.568	-0.140	1.439
	Kurtosis	5.413	7.273	23.824
	Correlation	0.628	0.858	0.955
SIZE×STR	Mean	0.050	0.810	1.680
	S.D.	4.420	5.930	8.500
	Skewness	-0.724	-0.090	1.274
	Kurtosis	5.614	7.339	21.448
	Correlation	0.630	0.869	0.964
SIZE×MOM	Mean	0.140	0.820	1.550
	S.D.	4.580	5.950	8.800
	Skewness	-2.544	-0.041	1.904
	Kurtosis	5.073	8.746	41.653
	Correlation	0.523	0.843	0.958
SIZE×LTR	Mean	0.550	0.910	1.290
	S.D.	4.330	5.870	8.420
	Skewness	-0.620	-0.058	1.729
	Kurtosis	5.052	7.874	21.217
	Correlation	0.632	0.873	0.953

B. Average Correlation

	SIZE×BM	SIZE×STR	SIZE×MOM	SIZE×LTR
SIZE×BM	0.846	0.853	0.839	0.854
SIZE×STR		0.857	0.845	0.858
SIZE×MOM			0.826	0.844
SIZE×LTR				0.856

Note: As the test portfolios, we consider the following four sets of 26 portfolios: the equal-weighted market portfolio plus (i) 25 size and book-to-market quintile combinations (SIZE×BM), (ii) 25 size and short-term-reversal combinations (SIZE×STR), (iii) 25 size and momentum combinations (SIZE×MOM), and (iv) 25 size and long-term-reversal combinations (SIZE×LTR). The table shows the minimum, median, and maximum of the means (annualized %), standard deviations (annualized %), skewness, kurtosis, and correlations of each set of 26 test portfolios (panel A) and the average correlations (panel B) of these four test portfolio classes. In panel B, the diagonal elements indicate the average of the correlations among the 26 test portfolio excess returns belonging to the corresponding firm characteristics class, while off-diagonal elements show the average of the correlations between two classes of 26 test portfolios belonging to the corresponding test portfolio classes. The sample period ranges from July 1937 to December 2013 for the SIZE×BM and SIZE×STR test portfolio sets, from January 1938 to December 2013 for the SIZE×MOM test portfolio sets, and from January 1950 to December 2013 for the SIZE×LTR test portfolio sets.

3.2. Basis Assets

The new factors are constructed from basis assets. To obtain high-quality new factors, it is necessary to find high-quality basis assets. The quality of the basis assets can be judged from at least two aspects: (1) how well the basis assets represent the investment opportunity set spanned by the asset universe and (2) the extent that the basis assets are not correlated with one another.

As a simple approach, Suh, Song, and Lee (2014) suggest finding basis assets among readily available, well established, and well diversified portfolios.[8] Specifically, to generate new pricing factors, we consider as the basis assets the following eight classes of firm characteristic portfolios: 10 size-sorted portfolios (SIZE), 10 book-to-market-sorted portfolios (BM), 10 earnings-to-price-sorted portfolios (EP), 10 cash-flow-to-price-sorted portfolios (CFP), 10 dividend-yields-sorted portfolios (DY), 10 short-term-reversal-sorted portfolios (STR), 10 long-term-reversal-sorted portfolios (LTR), and 10 momentum-sorted portfolios (MOM).[9] In total, we have 80 portfolios (i.e., 10 portfolios for each of eight classes). Because a typical out-of-sample exercise for portfolio selection is repeatedly conducted with relatively few data (for example, 60 months), we reduce the number of basis assets by selecting three portfolios (i.e., the first, fifth, and tenth deciles) from each firm-characteristic class. Therefore, the basis assets consist of 24 portfolios (i.e., three portfolios for each of eight classes). This approach of constructing the basis assets has at least three advantages. First, it can be easily implemented. Second, we can link pricing factors to be constructed from the basis portfolios with firm characteristics. Third, this method can readily accommodate new firm characteristics that can be found by future studies.

Table 2 shows summary statistics of the basis assets during the sample period ranging from July 1951 to December 2013.[10] The mean returns are differentiated among deciles for each firm characteristic class. All classes show a similar level of volatility, and most are negatively skewed. It is noted that high positive correlations are shown among not only basis assets belonging to the same firm characteristic class but also basis assets belonging to different classes. This fact may suggest the necessity of including many firm characteristics to successfully represent investment opportunities.

[8] According to Markowitz (1952), Cass and Stiglitz (1970), and Ross (1978), the investment opportunity set can be reduced to a group of portfolios. By appropriately producing such portfolios, we may obtain good basis assets to represent the opportunity set. Ahn et al. (2009) sort individual assets into portfolios using statistical cluster analysis, wherein firms are sorted by maximizing the correlation within a group and minimizing the correlation between groups. Despite its theoretical advantage, this method incurs high implementation costs due to its requirement of a large data set consisting of all individual assets.

[9] These data are also available at Kenneth French's web page. A large body of literature has documented many portfolio sorts based on firm or equity characteristics. For the motivations of these eight characteristics, refer to Banz (1981) for the size effect, Basu (1977) for the effect of the earnings-to-price ratio, Stattman (1980) and Rosenberg et al. (1985) for the effect of the book-to-market ratio, Chan et al. (1991) and Lakonishok et al. (1994) for the effect of the cash-flow-to-price ratio, Keim (1983) for the effect of the dividend yield, Fama (1965), Jegadeesh (1990), and Lehmann (1990) for a short-term-reversal effect, Carhart (1997) for a momentum effect, and Daniel and Titman (2006) for a long-term-reversal effect.

[10] The beginning time of our sample period is determined by the EP and CFP decile data availability.

Table 2. Summary Statistics of Basis Assets and Their Correlations

Decile	SIZE	BM	EP	CFP	DY	STR	MOM	LTR
				A. Mean				
1	0.825	0.527	0.514	0.508	0.606	0.734	-0.098	0.889
5	0.817	0.676	0.635	0.685	0.580	0.673	0.530	0.705
10	0.545	0.969	1.053	1.052	0.638	0.253	1.224	0.599
				B. Standard deviation				
1	6.053	4.993	5.511	5.390	5.603	6.911	7.582	6.256
5	5.357	4.307	4.365	4.464	4.564	4.375	4.313	4.280
10	4.157	5.710	5.217	5.160	4.416	5.300	5.962	5.746
				C. Skewness				
1	-0.161	-0.274	-0.258	-0.375	-0.439	-0.283	0.636	0.294
5	-0.517	-0.437	-0.397	-0.531	-0.392	-0.371	-0.300	-0.302
10	-0.375	0.076	-0.272	-0.347	-0.264	-0.264	-0.456	-0.398
				D. Kurtosis				
1	5.622	4.371	4.401	4.449	4.735	6.231	7.809	6.266
5	5.362	5.450	4.878	5.794	6.113	5.028	5.243	5.448
10	4.527	7.433	5.241	5.203	9.143	5.107	4.904	4.420
				E. Average correlation				
SIZE	0.908	0.843	0.827	0.825	0.786	0.850	0.818	0.838
BM		0.846	0.851	0.850	0.827	0.843	0.822	0.842
EP			0.836	0.850	0.826	0.840	0.819	0.836
CFP				0.834	0.824	0.838	0.818	0.836
DY					0.795	0.811	0.797	0.811
STR						0.842	0.825	0.837
MOM							0.795	0.812
LTR								0.824

Note: As the basis assets, we consider the following eight classes of characteristic portfolios: 10 size-sorted portfolios (SIZE), 10 book-to-market-sorted portfolios (BM), 10 earnings-to-price-sorted portfolios (EP), 10 cash-flow-to-price-sorted portfolios (CFP), 10 dividend-yields-sorted portfolios (DY), 10 short-term-reversal-sorted portfolios (STR), 10 long-term-reversal-sorted portfolios (LTR), and 10 momentum-sorted portfolios (MOM). The table shows the mean (annualized %, panel A), standard deviation (annualized %, panel B), skewness (panel C), kurtosis (panel D), and average correlations (panel E) of these eight basis asset classes. In panel E, the diagonal elements indicate the average of the correlations among the 10 basis assets belonging to the corresponding firm characteristics class, while the off-diagonal elements indicate the average of the correlations between two classes of 10 basis assets belonging to the corresponding firm characteristics classes. The sample period ranges from July 1951 to December 2013.

3.3. Pricing Factors

The new factors are formed from the 24 basis assets, consisting of three portfolios for each of eight firm characteristics.[11] As shown in Table 2, the basis assets exhibit high positive correlations. This feature justifies the use of PCA to reduce multiple firm

[11] For the earliest periods when the EP and CFP deciles are not available, the basis assets and the new factors are formed by including only available firm-characteristic portfolios.

characteristics into a small number of factors.

The new model includes one CS factor and multiple TS factors. Choosing K, the number of factors, is an important step and may significantly affect portfolio-selection performance. Instead of arbitrarily choosing K, we compare for the test portfolios the performances of prominent extant models with the new model by including one CS factor and varying the number of TS factors, thereby studying the effect of the number of the new factors on portfolio investment performances.

Table 3 shows summary statistics of the new CS and TS factors. The new factors are constructed to be uncorrelated to one another. Because the CS factor is determined after the determination of the TS factors, the single CS factor is dependent upon the choice of the number of TS factors. As more TS factors are included, the volatility of the last TS factor diminishes, which is understandable considering the PCA properties. Similarly, as more TS factors are included, the volatility of the CS factor also diminishes; however, it diminishes at a slower rate and maintains a comparable level to the last TS factor, demonstrating that the single CS factor maintains greater importance than the corresponding last TS factor.

Table 3. Summary Statistics of New Factors

Number	TS Factors				CS Factors			
of TSF	Mean	S.D.	Skewness	Kurtosis	Mean	S.D.	Skewness	Kurtosis
1	-6.042	39.321	0.488	5.102	2.538	5.621	-0.361	4.873
2	-0.250	7.746	0.838	9.446	2.498	5.479	-0.113	4.192
3	0.730	7.502	0.001	7.160	2.346	5.252	-0.305	5.152
4	1.369	5.997	-0.619	8.526	1.773	4.613	-0.012	3.471
5	-0.570	4.022	0.040	11.476	1.607	4.461	0.034	3.414

Note: The new factors are formed from 24 basis assets constructed by selecting three portfolios (i.e., the first, fifth, and tenth deciles) for each of eight characteristic classes. The table shows the summary statistics of the new factors for various TS factors. Note that the CS factor is determined after the determination of the TS factors; therefore, the single CS factor is dependent upon the number of TS factors. The sample period ranges from July 1951 to December 2013.

4. PORTFOLIO INVESTMENT PERFORMANCES

In this section, we provide the out-of-sample portfolio investment results from several popular models for comparison with the results of the new factor models by varying the number of the new factors. Portfolio investment performances are measured by the Sharpe ratio. In addition to the three prominent extant factor models described in subsection 2.2, we include a sample moment approach, which suggests estimating the mean μ and covariance matrix V in Eq. (8) as their sample counterparts. We also consider several popular passive portfolio-selection methods for comparison purposes: (i) the value-weighted market portfolio (VW), (ii) equal-weighted market portfolio (EW), and (iii) equal-weighted portfolio of the test portfolios $(1/\mathrm{n})$.

We include in the portfolio-selection methods several restrictions to assess whether they improve portfolio investment performance. For example, we impose a position limit, following MacKinlay and Pástor (2000). In particular, if at least one weight in the portfolio exceeds 50% in absolute value, the portfolio weights are scaled down such that the largest absolute weight equals 50% and the remaining funds are invested in the VW.[12] This restriction may be effective when a model produces extreme weights. Another restriction is imposed on the covariance matrix of the expected asset returns. MacKinlay and Pástor (2000) document evidence that imposing a simple structure on the covariance matrix may improve portfolio performance. For the sample moment approach, we consider the restriction of the identity covariance matrix, as done by MacKinlay and Pástor (2000). For the factor models, we consider the restriction of imposing a diagonal covariance matrix for disturbances. Lastly, we need to determine the estimation window. In particular, we consider 60 or 120 months, following MacKinlay and Pástor (2000).

4.1. Results for Test Portfolios

We illustrate the out-of-sample Sharpe ratios of the four factor models for the SIZE×BM test portfolios in table 4.[13] For robustness, the results are presented for not only the whole sample period but also sub-periods of approximately 20 years. Each factor model of the CAPM, FF3, and FF4 has eight variants by combining the position limit restriction, the covariance restriction, and the estimation window choice, whereas the new factor model has 40 variants (i.e., eight variants times the additional five choices of the number of the new factors from one to five). Interestingly, the position limit restriction does not provide an improvement for the CAPM but greatly improves the performances of the other three factor models. The unrestricted portfolio-selection methods perform very poorly.[14] Unlike the position limit restriction, the covariance restriction and the estimation window choice improve the results in some cases but not in others. The number of factors in the new factor model significantly affects the portfolio performance. In particular, too few or too many new factors do not deliver the best results in most cases. For the whole sample period, the FF3 delivers the highest Sharpe ratio (0.201) with the position limit restriction, the diagonal covariance restriction, and an estimation window of 120 months. The new factor model yields the

[12] Alternatively, we also attempt to impose a position limit such that the absolute weights are limited to 50% and the remaining portions are invested in the VW. We obtain similar results, which are omitted for brevity.

[13] For simplicity, we omit the results for the other three test portfolio sets, which are available upon request.

[14] This has been well documented by many previous studies, such as Dickinson (1974), Jobson and Korkie (1980), Michaud (1989), Best and Grauer (1991), Jorion (1991), Black and Litterman (1992), Green and Hollifield (1992), and Chopra and Ziemba (1993).

second-highest Sharpe ratio (0.190) with the position limit restriction, no diagonal covariance restriction, an estimation window of 60 months, and five new factors. The FF4 generates a Sharpe ratio of 0.183 and the CAPM a Sharpe ratio of 0.154, with both values being implemented with the position limit restriction, the diagonal covariance restriction, and an estimation window of 120 months. Understandably, the FF3 may perform the best among the factor models for the test portfolios SIZE×BM because the SMB and HML factors in the FF3 are designed to account for the effects of size and book-to-market ratio on which the SIZE×BM test portfolios are constructed by double sorts.

Table 4. Out-of-sample Sharpe Ratio of Factor Models for the SIZE×BM Test Portfolios

pos limit	diag	T	1937-2013	1937-1956	1957-1976	1977-1996	1997-2013
A. Factor Model: CAPM							
0	0	60	0.145	0.159	0.112	0.173	0.143
0	0	120	0.146	0.159	0.114	0.174	0.144
0	1	60	0.152	0.177	0.112	0.168	0.150
0	1	120	0.154	0.177	0.113	0.168	0.157
1	0	60	0.145	0.159	0.112	0.173	0.143
1	0	120	0.146	0.159	0.114	0.174	0.144
1	1	60	0.152	0.177	0.112	0.168	0.150
1	1	120	0.154	0.177	0.113	0.168	0.157
B. Factor Model: FF3							
0	0	60	0.043	0.054	0.070	0.067	-0.032
0	0	120	0.088	0.139	0.099	0.233	0.037
0	1	60	0.010	0.073	0.030	-0.073	-0.008
0	1	120	0.039	0.048	0.077	0.237	-0.045
1	0	60	0.165	0.173	0.175	0.162	0.146
1	0	120	0.182	0.198	0.151	0.253	0.147
1	1	60	0.167	0.200	0.168	0.152	0.142
1	1	120	0.201	0.212	0.158	0.267	0.179
C. Factor Model: FF4							
0	0	60	0.011	0.076	0.078	0.116	-0.067
0	0	120	-0.013	-0.050	0.137	0.122	-0.030
0	1	60	-0.027	-0.063	0.046	-0.021	0.091
0	1	120	-0.017	-0.089	-0.036	0.130	0.060
1	0	60	0.178	0.213	0.167	0.144	0.194
1	0	120	0.171	0.195	0.170	0.184	0.143
1	1	60	0.167	0.185	0.155	0.155	0.176
1	1	120	0.183	0.213	0.155	0.187	0.177

Table 4. Out-of-sample Sharpe Ratio of Factor Models for the SIZE×BM Test Portfolios (Con't)

pos limit	diag	T	nNF	1937-2013	1937-1956	1957-1976	1977-1996	1997-2013
				D. Factor Model: New factors				
0	0	60	1	0.074	0.070	0.063	0.122	0.050
0	0	60	2	-0.031	0.101	-0.057	-0.065	-0.066
0	0	60	3	-0.050	-0.025	-0.048	-0.081	-0.034
0	0	60	4	0.035	0.047	-0.022	0.091	0.004
0	0	60	5	0.044	0.100	-0.042	0.068	0.061
0	0	120	1	-0.016	-0.035	-0.022	-0.011	0.093
0	0	120	2	-0.031	-0.061	0.093	-0.040	-0.046
0	0	120	3	0.055	0.099	0.095	0.043	0.116
0	0	120	4	0.032	0.046	0.080	0.197	0.014
0	0	120	5	0.052	0.099	-0.007	-0.114	0.152
0	1	60	1	-0.034	-0.003	0.081	-0.039	-0.090
0	1	60	2	-0.029	-0.079	0.074	-0.088	0.002
0	1	60	3	-0.033	-0.070	0.119	-0.057	-0.047
0	1	60	4	0.002	0.003	0.131	-0.097	0.050
0	1	60	5	0.033	-0.012	0.037	0.065	0.109
0	1	120	1	0.028	-0.002	0.038	0.060	0.095
0	1	120	2	0.050	0.081	0.054	0.049	0.031
0	1	120	3	0.005	0.104	0.121	-0.034	0.079
0	1	120	4	0.027	0.025	0.121	0.120	0.069
0	1	120	5	0.032	0.038	0.037	-0.046	0.067
1	0	60	1	0.154	0.142	0.154	0.150	0.178
1	0	60	2	0.184	0.224	0.186	0.113	0.207
1	0	60	3	0.181	0.178	0.202	0.123	0.231
1	0	60	4	0.180	0.205	0.177	0.132	0.210
1	0	60	5	0.190	0.225	0.180	0.162	0.188
1	0	120	1	0.138	0.119	0.227	0.109	0.094
1	0	120	2	0.136	0.113	0.235	0.140	0.069
1	0	120	3	0.151	0.143	0.157	0.182	0.140
1	0	120	4	0.173	0.181	0.140	0.207	0.178
1	0	120	5	0.175	0.200	0.154	0.147	0.200
1	1	60	1	0.142	0.137	0.167	0.118	0.150
1	1	60	2	0.163	0.185	0.166	0.094	0.202
1	1	60	3	0.179	0.187	0.181	0.126	0.222
1	1	60	4	0.173	0.196	0.159	0.120	0.219
1	1	60	5	0.173	0.200	0.152	0.134	0.209
1	1	120	1	0.123	0.112	0.193	0.099	0.092
1	1	120	2	0.142	0.179	0.160	0.145	0.087
1	1	120	3	0.150	0.150	0.139	0.172	0.144
1	1	120	4	0.153	0.138	0.131	0.190	0.159
1	1	120	5	0.147	0.137	0.133	0.157	0.167

Note: This table shows the out-of-sample Sharpe ratio of the factor models for the SIZE×BM test portfolios. The factor models are CAPM (panel A), the Fama and French (1993) three-factor model (panel B), the four-factor model combining the Fama-French three-factor model and Carhart's (1997) momentum factor model (panel C), and the new factor model (panel D). A "pos limit" value of one indicates a position limit restriction. If at least one weight in the portfolio exceeds 50% in absolute value, the largest portfolio weight is limited to 50% and the remaining portion is invested into the value-weighted market portfolio. A "diag" value of one denotes the restriction of imposing a diagonal covariance matrix for disturbances. "T" denotes the estimation window. "nNF" indicates the number of the new factors. The sample period ranges from July 1937 to December 2013.

Table 5. Out-of-sample Sharpe Ratios for All Four Test Portfolios

Model	T	1937-2013	1937-1956	1957-1976	1977-1996	1997-2013
A. Test Portfolios: SIZE×BM						
VW Market		0.137	0.184	0.087	0.153	0.114
EW Market		0.151	0.174	0.110	0.167	0.154
1/n		0.151	0.162	0.118	0.179	0.153
S.M.	60	0.009	-0.064	-0.068	0.107	0.258
S.M.	120	-0.021	-0.056	0.076	-0.084	0.349
S.M. V=I	60	0.117	0.182	0.088	0.200	0.176
S.M. V=I	120	0.159	0.158	0.144	0.193	0.158
CAPM		0.165	0.180	0.138	0.188	0.154
FF3		0.194	0.205	0.180	0.238	0.154
FF4		0.170	0.161	0.161	0.216	0.146
NF		0.202	0.140	0.249	0.272	0.167
B. Test Portfolios: SIZE×STR						
VW Market		0.137	0.184	0.087	0.153	0.114
EW Market		0.151	0.174	0.110	0.167	0.154
1/n		0.144	0.169	0.103	0.165	0.137
S.M.	60	-0.005	-0.010	0.100	-0.034	0.045
S.M.	120	-0.056	-0.087	0.006	-0.073	0.121
S.M. V=I	60	0.080	0.155	0.025	0.023	0.152
S.M. V=I	120	0.065	0.225	0.145	0.048	0.145
CAPM		0.162	0.185	0.124	0.183	0.157
FF3		0.148	0.147	0.131	0.216	0.108
FF4		0.093	0.106	0.041	0.126	0.110
NF		0.163	0.129	0.206	0.168	0.162
Model	T	1938-2013	1938-1956	1957-1976	1977-1996	1997-2013
C. Test Portfolios: SIZE×MOM						
VW Market		0.136	0.182	0.087	0.153	0.114
EW Market		0.150	0.171	0.110	0.167	0.154
1/n		0.143	0.165	0.104	0.162	0.144
S.M.	60	0.127	0.021	0.217	0.182	0.226
S.M.	120	0.390	0.214	0.466	0.631	0.311
S.M. V=I	60	0.093	0.164	0.007	0.151	0.079
S.M. V=I	120	0.131	0.168	0.149	0.132	0.119
CAPM		0.161	0.176	0.123	0.181	0.166
FF3		0.130	0.189	0.037	0.139	0.152
FF4		0.226	0.222	0.285	0.207	0.195
NF		0.243	0.150	0.306	0.267	0.281

Table 5. Out-of-sample Sharpe Ratios for All Four Test Portfolios (Con't)

Model	T	1950-2013	1950-1956	1957-1976	1977-1996	1997-2013
D. Test Portfolios: SIZE×LTR						
VW Market		0.153	0.345	0.087	0.153	0.114
EW Market		0.166	0.312	0.110	0.167	0.154
1/n		0.180	0.308	0.122	0.185	0.168
S.M.	60	0.030	0.083	-0.065	0.053	-0.066
S.M.	120	0.105	-0.014	0.133	0.175	0.140
S.M. V=I	60	0.180	0.311	0.104	0.210	0.168
S.M. V=I	120	0.190	0.307	0.143	0.202	0.171
CAPM		0.190	0.314	0.134	0.203	0.164
FF3		0.207	0.353	0.112	0.207	0.238
FF4		0.173	0.372	0.126	0.187	0.100
NF		0.195	0.250	0.158	0.208	0.186

Note: This table shows the out-of-sample Sharpe ratios for all test portfolios. Three passive portfolio-selection models are considered: the value-weighted market portfolio (VW Market), equal-weighted market portfolio (EW Market), and equal-weighted portfolio of the test portfolios (1/n). The sample moment (S.M.) approach is also considered. "V=I" indicates the restriction of imposing an identity matrix instead of the sample covariance matrix. The factor models are CAPM, the Fama and French (1993) three-factor model (FF3), the four-factor model combining the Fama-French three-factor model and Carhart's (1997) momentum factor model (FF4), and the new factor model (NF). Among the multiple model specifications described in table 4, each factor model chooses at each point in time the best-performing specification for the previous period. "T" denotes the estimation window. The sample period ranges from July 1937 to December 2013 for the SIZE×BM and SIZE×STR test portfolio sets, from January 1938 to December 2013 for the SIZE×MOM test portfolio sets, and from January 1950 to December 2013 for the SIZE×LTR test portfolio sets.

Given the multiple model specifications illustrated in table 4, choosing a particular model specification ex ante is problematic. Moreover, because the determination of the model specification significantly affects the model performance, it is important to choose a good model specification ex ante. Instead of developing a formal framework, we rely on an ad hoc but simple rule.[15] We choose at each point in time the best-performing specification for the previous period among multiple model specifications.[16] This model choice rule is applied to the subset for which the position limit restriction is imposed. Table 5 reports the out-of-sample Sharpe ratios for all four test portfolio sets with the ex ante model choice rule. The results for the passive

[15] Parameter uncertainty has been well addressed within a Bayesian framework. See, for example, Zellner and Chetty (1965), Klein and Bawa (1976), Brown (1978), Kandel and Stambaugh (1996), and Barberis (2000). However, a formal framework has not yet been fully developed for model uncertainty arising from multiple restrictions or model specifications.

[16] Alternatively, we also choose the best- and second-best-performing specifications and take the average of the portfolio weights from the chosen two specifications. We obtain similar results, which are omitted for brevity.

portfolio-selection and sample moments method are also provided for comparison. The three passive portfolios perform quite well and are difficult to outperform, consistent with previous studies (for example, MacKinlay and Pástor (2000)). The identity covariance restriction on the sample moments method advocated by MacKinlay and Pástor (2000) works well in some cases but not in others. The FF3 may perform the best for the test portfolios SIZE×BM because the double sorts to generate the test portfolios are also taken into account by the FF3 factors. Interestingly, the new factor model performs better than the FF3 for the whole sample period. Similarly, the new factor model performs better than the FF4 for the test portfolios SIZE×MOM despite the FF4's explicit inclusion of the momentum factor. Overall, the new factor model outperforms the other methods in most cases for not only the whole sample period but also the sub-periods.

4.2. Summary of Portfolio Performance Comparisons

Table 6. Summary of Model Comparisons

Model	1937-2013	1937-1956	1957-1976	1977-1996	1997-2013
A. Average of the Sharpe ratio differences					
VW Market	0.060	-0.056	0.143	0.076	0.085
EW Market	0.046	-0.041	0.120	0.062	0.046
1/n	0.046	-0.034	0.118	0.056	0.049
S.M.	0.161	0.160	0.184	0.152	0.084
S.M.	0.096	0.153	0.060	0.067	-0.031
S.M. V=I	0.083	-0.036	0.174	0.083	0.056
S.M. V=I	0.064	-0.047	0.084	0.085	0.051
CAPM	0.031	-0.047	0.100	0.040	0.039
FF3	0.031	-0.056	0.115	0.029	0.036
FF4	0.035	-0.048	0.077	0.045	0.061
B. Number of outperformances by the new factor model					
VW Market	4	1	4	4	4
EW Market	4	1	4	3	4
1/n	4	1	4	3	4
S.M.	4	4	4	4	2
S.M.	4	4	3	3	2
S.M. V=I	4	1	4	3	3
S.M. V=I	4	2	4	3	3
CAPM	3	1	4	3	4
FF3	3	1	4	2	1
FF4	4	1	4	3	3

Note: This table shows the average of the out-of-sample Sharpe ratio differences between the new factor model against alternative models for all test portfolios (panel A) and the number of outperformances by the new factor model against alternative models for four test portfolio sets (panel B). Please refer to the description of table 5 for more information.

To facilitate model comparison, Table 6 compares the out-of-sample Sharpe ratios of the new factor model against alternative models for the four test portfolio sets. In particular, Table 6 provides the averages of the Sharpe ratio differences between the new factor model against the alternative models and the number of outperformances by the new factor model against alternative models for the four test portfolio sets.

The average out-of-sample Sharpe ratio of the new factor model is greater than those of all alternative models for the whole sample period. The relative gain of the new factor model is greatest against the sample moments methods, followed by the passive portfolios. For the three alternative factor models, the new factor model shows a similar level of gains. The new factor model outperforms alternative models for not only the whole sample period but also most sub-periods except for the first sub-period between 1937 and 1956.

We obtain similar results for the number of outperformances. Out of the four test portfolio cases, the new factor model outperforms the CAPM and the FF3 in three cases and the other models in all four cases for the whole sample period. As for the above-average out-of-sample Sharpe ratio analysis, the number of outperformances for the whole sample period largely hold for most sub-periods except for the first sub-period between 1937 and 1956. In summary, the new factor model outperforms the alternative models in most cases.

4.3. Effects of Model Specifications

The aforementioned results illustrate that various model specifications significantly affect portfolio investment performance. To better understand the effects of model specifications, table 7 presents the mean effects of each model specification, keeping the other specifications constant over all test portfolio sets. The position limit restriction greatly improves the performances of the FF3, FF4, and new factor model, with similar levels of improvements. On the other hand, the diagonal covariance restriction does not significantly affect performance on average. Compared to a short estimation window (i.e., 60 months), a longer window (i.e., 120 months) tends to yield better performances. If we have no preferred model specification, the ex ante simple rule of model specification choice greatly improves performance relative to the average performances of the alternative specifications. In particular, the improvements are the greatest for the new factor model and the lowest for the CAPM. Compared with even the ex post best among alternative model specifications, the ex ante rule generates slightly better performance for the CAPM and the new factor model and only slightly worse performance for the FF3 and the FF4 model. The ex ante rule might yield better performances than the ex post best-performing specification when the best-performing specification changes over time but tends to persist. The number of the new factors significantly affects the performance of the new factor model. In particular, the new factor model with three factors produces the best performance in most cases.

Table 7. Effects of Model Specifications

Model	1937-2013	1937-1956	1957-1976	1977-1996	1997-2013
A. Position limit restriction vs. no restriction					
CAPM	0.000	0.000	0.000	0.000	0.000
FF3	0.137	0.142	0.084	0.119	0.138
FF4	0.146	0.139	0.107	0.097	0.114
NF	0.138	0.140	0.116	0.116	0.135
B. Diagonal covariance restriction vs. no restriction					
CAPM	0.005	0.009	-0.002	-0.003	0.012
FF3	0.005	0.013	-0.010	0.002	0.012
FF4	0.001	-0.039	-0.006	0.009	0.013
NF	-0.003	0.007	-0.003	-0.005	-0.004
C. Estimation window T: 120 vs.60					
CAPM	0.000	-0.001	0.001	0.000	0.000
FF3	0.001	-0.007	0.010	0.008	0.014
FF4	0.022	0.023	0.006	0.064	0.009
NF	0.004	-0.023	0.038	0.018	0.025
D. Model specification choice: ex ante rule vs. ex post best performance					
CAPM	0.010	0.002	0.018	0.013	0.001
FF3	-0.002	-0.008	-0.020	-0.005	-0.012
FF4	-0.012	-0.013	-0.025	-0.015	-0.034
NF	0.022	-0.078	0.002	0.038	-0.041
E. Model specification choice: ex ante rule vs. ex post average performance					
CAPM	0.014	0.008	0.021	0.016	0.008
FF3	0.088	0.083	0.051	0.106	0.079
FF4	0.087	0.089	0.072	0.067	0.053
NF	0.125	0.087	0.133	0.161	0.089
F. Average out-of-sample Sharpe ratio of the new factor model along with the number of new factors					
nNF = 1	0.064	0.053	0.090	0.055	0.105
nNF = 2	0.075	0.093	0.086	0.064	0.080
nNF = 3	0.091	0.091	0.131	0.091	0.114
nNF = 4	0.083	0.094	0.118	0.088	0.104
nNF = 5	0.084	0.096	0.110	0.077	0.107

Note: This table shows the effects of various model specifications: the effects of position limit restriction (panel A), diagonal covariance restriction (panel B), estimation window choice (panel C), ex ante rule of model specification choice (panel D and E), and the number of the new factors for the new factor model (panel F). The effects are measured as the mean difference of the Sharpe ratios from panel A to E for all of the test portfolio sets. "nNF" in panel F indicates the number of the new factors in the new factor model. Please refer to the description of table 5 for more information.

5. CONCLUSION

In this paper, we apply the new factor model proposed by Suh, Song, and Lee (2014) to portfolio-selection problems and compare its portfolio investment performance with that of other popular portfolio-selection methods. The new factors are formed from a well-characterized subset of the asset universe called basis assets. These basis assets are structured based on the firm characteristics found by previous studies to be useful for explaining (co)variation in asset returns. Suh, Song, and Lee (2014) provide empirical results that the new factors exhibit better asset-pricing performance than popular extant asset-pricing factors.

Performance comparison results show that the new factors provide better portfolio investment performance than alternative methods for various test portfolios and various periods. However, some caveats are in order. Not only factors but also other model specifications affect performance. Our empirical results show that various model specification choices beyond the choice of factors also greatly affect portfolio investment performance. Although we propose a simple rule for model specification choice that proves to work well in many cases, more efficient methods should be developed to improve portfolio management.

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